

# Finite Element Analysis of Contact and Bending Stresses in Helical Gear Pair

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**Abstract:** In gear design, excessive tooth contact stresses and bending stresses are one of the prime gear failure factors; therefore, its analysis is very important in order to shorten the possibility of gear tooth failure. In the present work, the tooth bending stresses and contact stresses in a helical gear pair is calculated using AGMA theory and finite element analysis (FEA). The modelling of helical gear pair is carried out in CREO and ANSYS is used for FEA. It is observed that the bending stresses and contact stresses, both decreases with an increase in the helix angle if pressure angle remains constant. However, the error in the calculation by AGMA and FEA is higher for the bending stresses than the contact stresses and bending stresses.

**Key words:** Helical gear pair, Bending stresses, Contact stresses

## 1. INTRODUCTION

Gears are used in automobile, marine vessel, industrial equipment, etc. to transmit power and motion between the shafts by successively engaging teeth [1]. Generally, the gear failure is caused either due to the excessive bending stresses or contact stresses generated during the power transmission. Helical gears, in which teeth are cut at certain angle (helix angle), are preferred at high speeds due to the smooth operation and reduced noise during the power transmission. Gears transmit motion, without connector or intermediate link, by direct contact. The tangential contact is made between the surfaces of mating gears, which have either rolling/sliding motion along the point of contact [1]. When two bodies, having curved surfaces, are pressed together, the tangential contact changes the contact area resulting into the development of three-dimensional stresses.

The contact stresses develop when each meshing body has a double radius of curvature, i.e., when radius in a perpendicular plane is different from the radius in the plane of rolling [2]. Due to the pitting phenomenon, the gear tooth failure occurs when the contact stresses between two meshing teeth exceeds the material's surface endurance strength [3]. Several authors have investigated analytically/numerically the contact stresses in helical and spur gear pairs. The design of gear depends on the material properties and geometry of gear tooth profile because they fulfil, imply and indicate the functional requirements. Few authors have used composite materials in order to achieve longer gear life, less tooth failure, less weight [4, 5, 6]. The gear design criteria need to maximize the desirable impact, e.g., life of gear, potential reliability and minimize undesirable impact, e.g., backlash. The objective of present work is to investigate the effects of helix angle on the bending stresses and contact stresses in helical gear pair based on the AGMA (American Gear Manufacturing Association) theory and Finite Element Analysis (FEA).

## 2. BENDING STRESSES

Tooth bending stresses is important parameter for the design of gear pair. Several authors used either Lewis equation or AGMA theory for estimating the bending stresses in a gear pair. The maximum bending stresses is observed at the root of gear tooth. AGMA theory defines the bending stresses by the following relationship [2]

$$\sigma_b = \frac{F_t}{bmj} K_o K_v (0.93 K_m)$$

Where  $F_t$  the tangential load,  $m$  the module,  $j$  the geometry factor for bending stress and  $b$  the face width. Table 1 shows the magnitude of bending stresses based on the AGMA theory.

**Table 1: Bending stresses based on AGMA**

Helix Angle	Geometry Factor	Bending Stress (MPa)
15°	0.4987	111.72
20°	0.5005	111.32
25°	0.5031	110.74
30°	0.5066	109.97
Pressure angle = 20°		

## 3. TOOTH CONTACT STRESSES

The tooth contact stresses is another criterion for the safe design of helical gear pair. The reduction in contact stresses reduces the level of noise during operation. Several authors have used 2D model for the contact stresses analysis of helical gear pair

based on the analytical equation or FEA. Table 2 shows a brief summary of contact stresses analysis of helical gear pair carried out by the different authors.

**Table 2: Review of contact stresses analysis for helical gear pair**

S. N.	Objectives	Methodology	Conclusions	Author / References
1.	Determination of maximum contact lowest point	AGMA theory & FEM	maximum contact stresses are observed at the lowest point of tooth	Seok-Chul Hwang [7]
2.	Determination of stress at constant co-efficient of friction		Stresses decreased with the helix angle and friction coefficient	Santosh S. Patil [8]
3.	To find stresses in high speed helical gear		Teeth failure is mainly due to the bending stress	J. Venkatesh [5]
4.	To find contact stress for different helix angles, face width and pressure angles		Face width, pressure angle and helix angle are important parameter for contact stresses	S. Sai Anusha [9]
5.	To find bending stresses at root of gear and contact stresses		Less error between the theoretical and FEA	Sathyanarayana [10]
6.	Effect of face width on contact stress		Almost identical contact stresses magnitude from AGMA and FEA	HlwanHtetHtet San [11]
7.	Effect of helix angle on helical gear pair	Hertz & AGMA theory	Minimum difference of 0.19 % between AGMA and ANSYS results	R. Devraj [12]
8.	To find distribution of bending stresses, contact stresses and torque transmission		Only 1-2% error in contact stresses from FEA and Hertzian methods	M. Gh. Khosrosha [13]
9.	Modification and analysis of involute gear	Modeling & Simulation	Mismatch in pinion tooth surface of helical gear in longitudinal direction.	Faydor L. Litvin [14]
10.	To find bending and contact stresses in gear pair under different loads		Aluminium silicon carbide and grey cast iron are found to be suitable	K. Naresh [15]
11.	Structural analysis of high speed helical gear used for marine engines	Lewis equation & FEM	Aluminium alloy reduces weight up to 55%.	B. Venkatesh [16]
12.	Helical gear tooth contact modelling with tooth deflection	Pseudo Interface Method & FEM	Reasonable distribution of tooth contact force along the line of action may be generated by using flexible teeth and flexible tooth foundation	Juha Hedlund [17]
13.	To find deformation and stresses in helical gears	Numerical Method	The maximum contact stress exists at the contact line position that lies at period with three contact position	Ivana Atana [18]
14.	Fatigue analysis of helical gear	Experimentation	Pitting due to fatigue and misalignment causes failure of tooth	Osman Asi [19]
15.	To predict wear in helical gear	Modeling & Simulation	Transmission error decrease with wear and sliding clearance is important	Anders Flodin [20]
16.	Fatigue wear reduction through micro-geometry modification	Non-linear finite element method	Micro geometry improves the surface fatigue wear	K. Mao [21]
17.	Position error estimation of gear caused by the surface mismatch	Computerised Simulation	Position error caused by surface mismatch and maximum stresses occur at tooth root	F. L. Litvin [22]
18.	To study effect of gear parameters on dynamic tooth load and tangential force	Buckingham equation & FEM	Dynamic tooth load and tangential force depends up on the helix angle	B. Venkatesh [23]

## 2.1 Analytical Method

The analytical methods are used for estimating the bending stresses, tooth contact stresses and surface fatigue. Mostly, Hertzian and AGMA theories are used for calculating the contact stresses. The Hertzian theory is limited to the calculation of contact stresses for frictionless surface having smaller contact area; therefore, AGMA theory is generally preferred.

### AGMA Theory

AGMA theory defines the contact stresses as [2]

$$\sigma_c = C_p \sqrt{\frac{F_t}{bd_p I} \left( \frac{\cos \Psi}{0.95 C_R} \right) K_o K_v (0.93 K_m)} \quad (1)$$

where  $d_p$  the pitch circle diameter of pinion.

The elastic coefficient factor ( $C_p$ ), geometry factor ( $I$ ), contact ratio ( $C_R$ ) may be expressed as [2]

$$C_p = \sqrt{\frac{1}{\pi \left( \frac{1-\nu_p^2}{E_p} + \frac{1-\nu_g^2}{E_g} \right)}} \quad (2)$$

$$I = \frac{\sin \phi \cos \phi}{2} \frac{i}{i+1} \quad (3)$$

$$C_R = \left( \frac{\sqrt{(r_1+a)^2 - r_{b1}^2} + \sqrt{(r_2+a)^2 - r_{b2}^2} - (r_1+r_2) \sin \phi}{\pi m \cos \phi} \right) \quad (4)$$

S Jyothirmai et al. [24] used AGMA theory for estimating the contact stresses and fatigue stresses in order to investigate the overall performance of gear pair in terms of tooth strength, stresses generated at low speeds under small loads. Seok-Chul Hwang et al. [7] used AGMA theory for calculating the contact stresses at lower point and higher point of teeth in contact. Other authors have used this theory for the contact stresses analysis of helical gear pair [7, 8, 9]. The contact stresses, based on Hertz and AGMA equations, decreases with an increase in the helix angle, assuming negligible co-efficient of friction [8]. B. Venkatesh et al. [16] used AGMA theory for the structural analysis of helical gear and found small difference with FEA. They concluded that suitable material may reduce the weight of gear pair. Table 3 gives the calculation of contact stresses in helical gear pair based on the AGMA theory

**Table 3: Contact stresses based on AGMA**

Helix Angle	Pinion diameter (mm)	Gear diameter (mm)	Geometry factor	Contact ratio	Contact Stresses (MPa)
15°	74.54	298.15	0.1320	1.5919	631.86
20°	76.62	306.48	0.1347	1.5317	620.26
25°	79.44	317.77	0.1383	1.4558	605.56
30°	83.14	332.55	0.1429	1.3655	587.89
For 20° pressure angle					

## 2.2 Finite Element Analysis

### Modelling

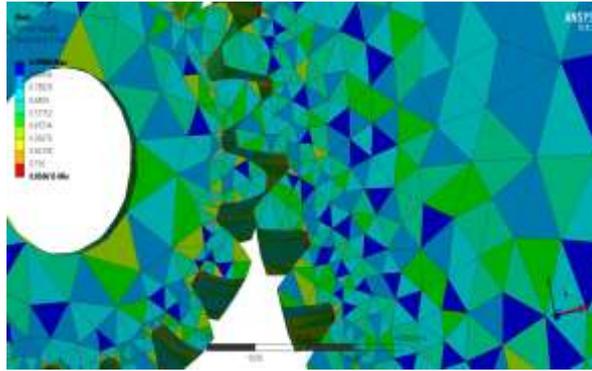
3D models are most effective for designing and improving the accuracy in gear design. The 3D model geometries are accurate and enable us to check and evaluate the gear operation. Ivana Atanasovka et al. [18] carried out FEA to determine the position of contact line during the contact period. CH Rama and Mohana Rao [25] calculated the helical gear root stresses in a more realistic manner. Litvin et al. [14] used 3D model for double circular arc helical gear for calculating the position error caused by the teeth surface mismatch. The helical gear pair is modelled in CREO software. Table 4 shows the modelling parameters for the helical gear pair.

**Table 4: Modelling parameters for helical gear**

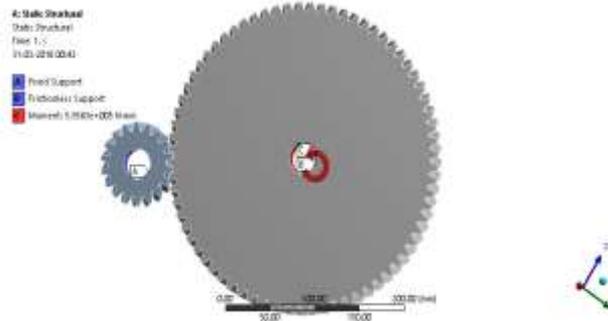
Gear Parameters	Value
Number of teeth on gear	72
Number of teeth on pinion	18
Gear diameter	332.64mm
Pinion diameter	83.2 mm
Pressure angle	20°
Module	4
Face width	30.16 mm
Helix angle	30°
Poisson ratio	0.3
Addendum	1m <sub>n</sub>
Dedendum	1.25m <sub>n</sub>

### FE Analysis

FEA is used for solving complex engineering problem when shape or applied load is complex. Several authors have used [26, 27, 28, 4, 29] FE stress analysis for gear pair. Shalini Keshari et al. [27] used FEA for optimization of the gear weight based on genetic algorithm (GA) and particle Swarna optimization technique. In the present work, the meshes are generated using triangular element (22362) with element length of 1mm to 10 mm. Fig. 1 shows the mesh generated in the helical gear drive. Fig. 2 shows the boundary conditions used in FEA in which helical gear is assumed to provide fixed support whereas frictionless support and moment load is applied on the mating gear (pinion). The magnitude of contact stresses are calculated by assuming helical gear of annealed AISI 8620 alloy steel with a tensile strength 530MPa and AISI 9310 alloy steel with a tensile strength 910MPa for the pinion. Poisson's ratio 0.3 is considered.



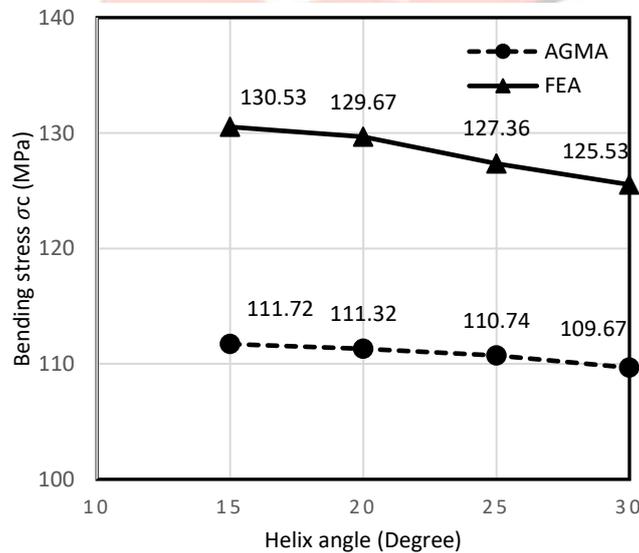
**Fig 1: Mesh Element**



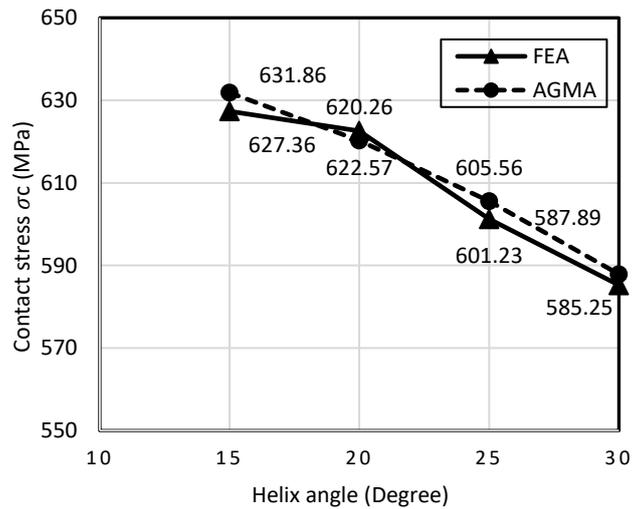
**Fig 2: Boundary conditions in helical gear pair**

**4. RESULTS AND DISCUSSION**

The bending stresses and contact stresses are calculated for different values of helix angles (15°, 20°, 25° and 30°). Fig. 3 shows the variation of bending stresses at different helix angles, keeping the pressure angle fixed at 20°. It can be observed that the bending stresses decreases gradually with an increase in helix angle. The error in bending stresses between AGMA and FEA is maximum (16.4%) for 15° helix angle; however, it is minimum (14.3%) for 30° helix angle. Fig. 4 shows the variation of contact stresses at different helix angles, keeping the pressure angle fixed at 20°. The contact stresses also decrease with an increase in the helix angle but the error between AGMA and FEA is very small (0.74%) for 25° helix angle.



**Fig. 3: Effect of helix angle on bending stresses**



**Fig. 4: Effect of helix angle on contact stresses**

## 5. CONCLUSION

Both bending stresses and contact stresses in helical gear pair depends on the helix angle, material and face width of gear. In the present work, the bending stresses and contact stresses are calculated using AGMA theory and FEA. Both stress decreases with an increase in the helix angle. It is concluded that the error in the estimation of contact stresses using AGMA theory and FEA is approximately 0.7% for the pressure angle at 20° and error in the estimation of bending stresses is 16.4% for pressure angle at 20°

## NOMENCLATURE

$b_0$	Contact area
$l$	Geometry factor
$F$	Component of force
$K_B$	Rim thickness factor
$K_A$	Application factor
$K_l$	Load factor
$C_f$	Surface condition factor
$K_r$	Reliability factor
$K_T$	Temperature factor
$K_v$	Dynamic factor
$K_s$	Surface factor
$C_p$	Elastic coefficient
$K_m$	Load distribution factor
$K_o$	Overload factor
$P_a$	Axial pitch
$d_p$	Pinion pitch diameter
$\Psi$	Helix Angle
$m_n$	Normal Module
$\phi$	Pressure Angle
$J$	Factor for Geometry
$K_s$	Size factor
$P_d$	Diametral pitch
$E$	Elastic module
$W^t$	Tangential tooth load
$\nu$	Poisson ratio

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