© IJEDR 2018 | Volume 6, Issue 3 | ISSN: 2321-9939

Study of System Uncertainty for Nonlinear Dynamic System

[1]Mohd Aqib, [2]Gulab Chandra Yadav
Electrical Engineering Department

Abstract—This paper presents the different types of system uncertainty for nonlinear dynamic system. System uncertainty are the principle factors responsible for degraded stability and performance of a nonlinear dynamic system. Uncertainty may take many forms but among the most significant are noise/disturbance signals and transfer function modeling errors. Uncertainty in any form is no doubt the major issue in most control system, we have analyze these uncertainty to represent how it affects the input/output relationship of the nonlinear system and by using Linear Fractional Transformations, these uncertainty are separated out from the nonlinear system model.

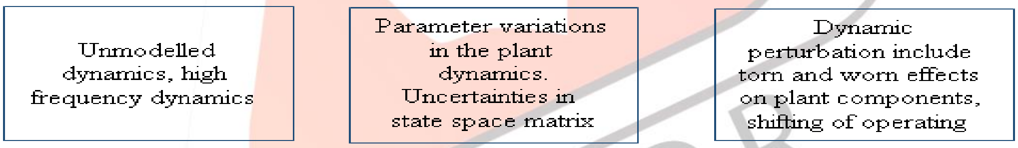
Index Terms—Nonlinear Dynamic System, Uncertainty, Linear Fractional Transformations.

I. INTRODUCTION

Uncertainty means which is not certain in terms of time and space. Its means lack of some kind of information for example in the case of mass-spring-damper system, mass can have some uncertainty in its volume. Spring have uncertainty due to aging effect. The resistance of a resistor can change due to its temperature which may cause uncertainty. So the practical system have definitely some kind of uncertainty. Many types of uncertainty are define like uncertainty due to randomness, due to vagueness, due to noise in signal etc.

In the case of nonlinear dynamic system; three types of uncertainty is observed

1. Unstructured uncertainty
2. Structured uncertainty
3. Parametric uncertainty



a) Unstructured uncertainty b) Structured uncertainty c) Parametric uncertainty

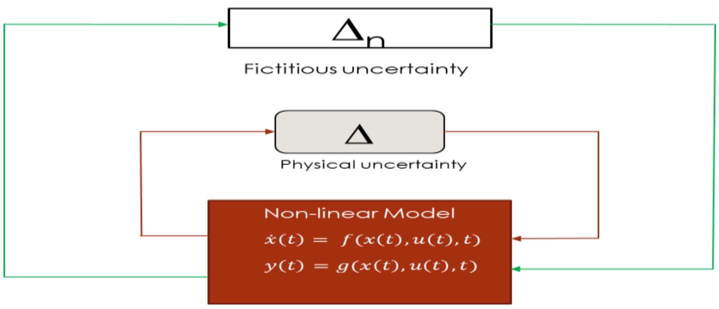
Fig. 1 Types of uncertainty

II. UNCERTAINTY IN NONLINEAR DYNAMIC SYSTEM [1]

a) Unstructured uncertainty:

Many dynamic perturbations that may occur in different part of a system can be lumped into one single perturbation block Δ (Some unmodelled, high-frequency dynamics). This uncertainty representation is referred to as 'unstructured' uncertainty.

In the case of non-linear dynamic system, the block Δ may be itself be a matrices and represent a different kind of physical uncertainty. Unstructured uncertainty usually represents frequency-dependent element such as actuator saturation and unmodelled structural modes in the high frequency range (or) plant disturbances in the low frequency range.



$$\dot{x}(t) = f_p(x(t),u(t),t) = (N+\Delta_N)\ (M+\Delta_M)^{-1}$$

Fig. 2 Unstructured uncertainty

b) Structured uncertainty [2]

Structured uncertainty represents parametric variation in the plant dynamics, for example:

1. Uncertainties in certain entries of state-space matrices (A, B, C, D) ,for example, the uncertain variation in an aircraft's stability and control derivatives.
2. Uncertainties in specific poles and/or zeroes of the nonlinear plant transfer function.
3. Uncertainties in specific loop gains/phases.

Nonlinear system with uncertainty can be described by:

$$\dot{x}(t) = f(x(t), u(t), w(t), \boldsymbol{\gamma}(t))$$
$$y(t) = g(x(t), u(t), w(t), \boldsymbol{\gamma}(t))$$

where*x(t)* is state vector ; *u(t)* is input vector ; *y(t)* is output vector ;*w(t)* is disturbance input vector ; ***γ****(t)* is uncertain vector

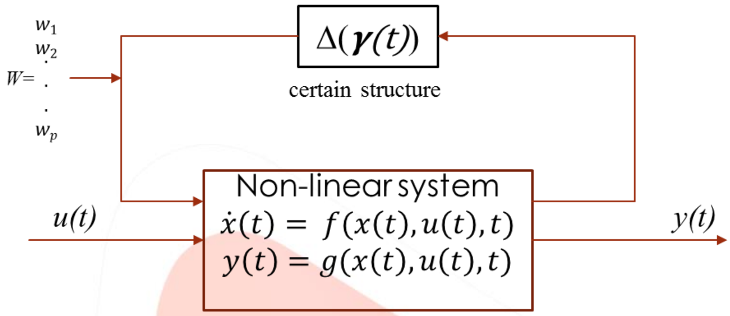


Fig. 3 Structured uncertainty

c) Parametric uncertainty [2]

The unstructured uncertainty is useful in describing unmodelled (or) neglected system dynamics. These complex uncertainties usually occur in the high-frequency range and may include unmodelled lags (time delay), parasitic coupling, hysteresis and other nonlinearities.Dynamic perturbations in many industrial control systems may also be caused by inaccurate description of component characteristics, torn-and-worn effects on plant components, (or) shifting of operating pts. Such perturbation may be represented by variation of certain system parameters over some possible value range (complex or real).

III. CASE STUDY

In this section we analyze the uncertain Mass-Spring-Damper System. Here we separate out the uncertainty block from parameter of the given system.

Mass-Spring-Damper System:

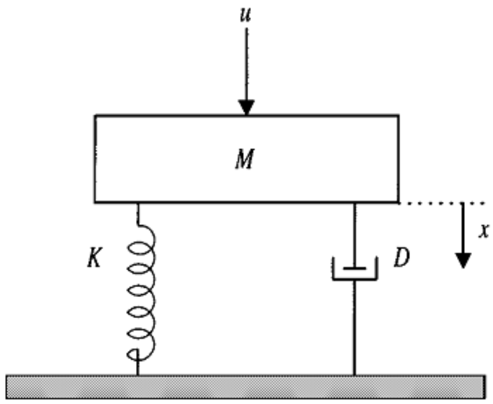


Fig. 4 Mass-Spring-Damper System

Its dynamic equation is given by

$M\ddot{x}(t) + g\big(x(t), \dot{x}(t)\big) + f\big(x(t)\big) = \emptyset(\dot{x}(t)u(t)$

where M is the mass and u is the force; $f\big(x(t)\big)$ depicts the spring nonlinearity & uncertainty; $g\big(x(t), \dot{x}(t)\big)$ depicts the damper nonlinearity; $\emptyset(\dot{x}(t)$ depicts the input nonlinearity & uncertainty.

$$g(x(t), \dot{x}(t)) = D(c_1 x(t) + c_2 \dot{x}(t)^3 + c_3(t)\dot{x}(t))$$
$$f(x(t) = k(c_4 x(t) + c_5 x(t)^3 + c_6(t)x(t))$$
$$\emptyset(\dot{x}(t) = 1.4387 + c_7 \dot{x}(t)2 + c_8(t)\cos(5\dot{x}(t))$$

The operating range of the states is assumed to be within -1.5 and 1.5. The parameters are chosen as follows:
M = D = K =1.0; C1 =0; C2 =1; the nonlinear system then becomes:

$$\ddot{x}(t) = -\dot{x}(t)^3 - 0.01x(t)^3 - c_3(t)\dot{x}(t) - c_6(t)x(t) + (1.4387 - 0.13\dot{x}(t)^2 + c_8(t)\cos(5\dot{x}(t)))u(t)$$

The nonlinear dynamic equation of Mass-Spring-Damper System is linearize as follows:
$\cos(5\dot{x}(t)) \cong 1;\ \dot{x}(t)^3 \approx 0$
Then nonlinear equation becomes linear time varying second order differential equation as given below:

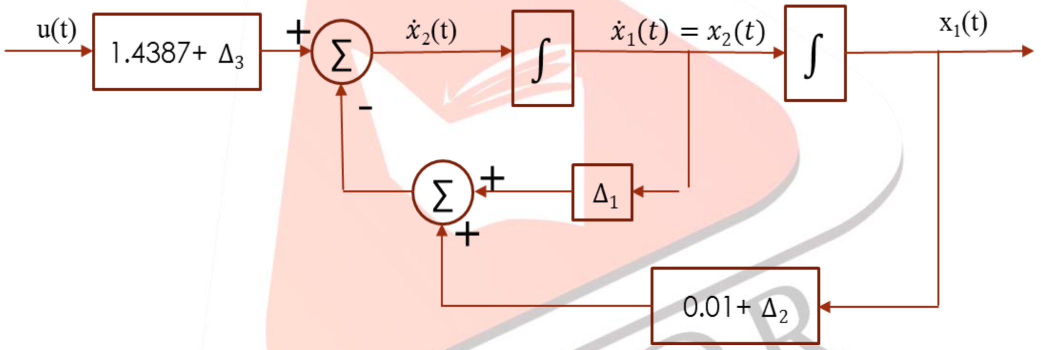
$$\ddot{x}(t) = -(0.01 + c_6(t))x(t) - c_3(t)\dot{x}(t) + (1.4387 + c_8(t))u(t)$$

Now let $c_3(t)=\Delta_1$; $c_6(t)=\Delta_2$; $c_8(t)=\Delta_3$; $x_1(t)=x(t)$; put these value in above we get following state space equation

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -(0.01 + \Delta_1)x(t) - \Delta_2 x_2(t) + (1.4387 + \Delta_3)u(t)$$

The above equation can realize as simulation diagram as follows:



Block diagram of given state equation

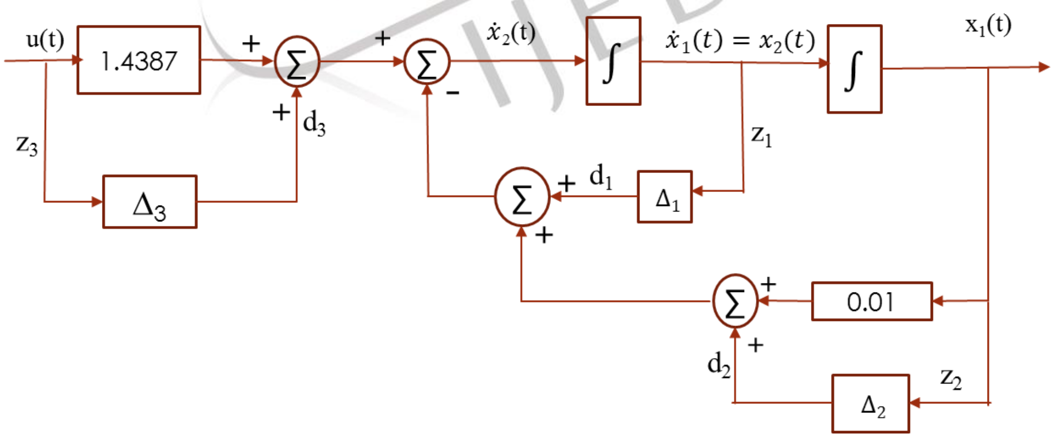


Fig. 5 Structured uncertainties block diagram

Let $z_1(t)$, $z_2(t)$, $z_3(t)$ be $x_2(t)$, $x_1(t)$, u(t) respectively, considered as another output vector; and d_1, d_2, d_3 be the signals coming out from the perturbation blocks as shown in above figure.

State space model is given as below:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -0.01 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0 & 0 & 0\\ -1 & -1 & -1\end{bmatrix}\begin{bmatrix}d_1\\ d_2\\ d_3\end{bmatrix} + \begin{bmatrix}0\\ 1.4387\end{bmatrix}u(t)$$

$$y(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

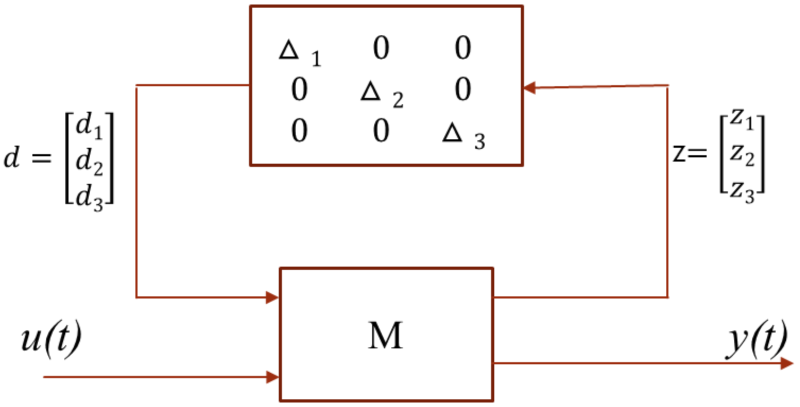


Fig. 6 Final block diagram

IV. Conclusion

We have apply Linear Fractional Transformations to mass-spring-damper system as an example of case study. The uncertainties in the system is separated as a separate block as shown in figure in case study section. It is concluded that Uncertainty affects decision-making and appears in a number of different forms and while linearizing the nonlinear model, the uncertainty are introduced in the model. It is observed that uncertain nonlinear dynamic system can be represented in M-Δ configuration by which study of uncertainty becomes easy.

References

[1] D.-W. Gu, P.Hr. Petkov and M.M. Konstantinov; Robust control design with Matlab"ch-2, "Modelling of Uncertain Systems", page no. 13-23; Springer,2005.

[2] Henning U. Voss and Jens Timmer, JurgenKurths; "Nonlinear dynamical System Identification from Uncertain and Indirect measurement " International Journal of Bifurcation and Chaos, Vol. 14, No. 6 (2004) 1905-1933.

[3] J. K. Hedrick and A. Girard,"Control of Nonlinear Dynamic Systems: Theory and Applications" 2010.

[4] J. Espinosa, J. Vandewalle, and V. Wertz, Fuzzy Logic, Identification and Predictive Control. Springer,2005.

[5] H. K. Lam, F. H. F. Leung, and P. K. S. Tam, "Stable and Robust Fuzzy Control for Uncertain Nonlinear Systems", IEEE Transaction on, Man, and Cybernetics—Vol. 30, No. 6, NOVEMBER 2000.

[6] Richard Y. Chiang,Michael G. Safonov, 'Robust Control ToolboxUser's Guide', "The robust control problem, page no.1-12",1984 - 1997 by The MathWorks, Inc.