Internal heating and Soret effect on Darcy - Brinkman convection in a binary viscoelastic fluid saturated porous layer

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***Abstract*—Linear and nonlinear analyses have been done and the combine effect of internal heating and Soret effect on Darcy - Brinkman convection in a binary viscoelastic fluid saturated porous layer, heated and salted from below, has been studied, analytically. Linear stability analysis has been performed by using normal mode technique and nonlinear analysis is done using truncated Fourier series. The modified Darcy-Brinkman-Oldroyd model, including the time derivative term, is employed for the momentum equation. The effects of Darcy number, Soret parameter, relaxation and retardation parameters, solute Rayleigh number, internal heat source, Lewis number and Darcy-Prandtl number on stationary and oscillatory convection are shown graphically. Also heat and mass transports are calculated in terms of the Nusselt number and Sherwood number and presented graphically.**

***Index Terms*— Double diffusive convection; Viscoelastic fluid; Internal heat source; Soret parameter; Porous media.**

I. INTRODUCTION

There is large number of practical situations in which convection is driven by internal heat source in a porous medium. The wide applications of such convection occur in nuclear reactions, nuclear heat cores, nuclear energy, nuclear waste disposals, oil extractions, and crystal growth. The study concerning internal heat source in porous media is provided by Tveitereid [1], who obtained the steady solution in the form of hexagons and two dimensional rolls for convection in a horizontal porous layer with internal heat source. Horton and Rogers [2] and Lapwood [3] were the first to obtained analytically the expression for critical Rayleigh number for the onset of convection in a fluid-saturated porous layer heated from below. Bejan [4] studied analytically the buoyancy induced convection with internal heat source, Parthiban and Patil [5] studied the effect of non-uniform boundaries temperature on thermal instability in a porous medium with internal heat source and predicted that internal heat source parameter advances the onset of convection. Hill [6] performed linear and nonlinear analyses on the double-diffusive convection in a porous layer with a concentration based internal heat source. Bhadauria et al. [7]-[8] studied effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium and also natural convection in a rotating anisotropic porous layer with internal heat source. Khan and Aziz [9] studied transient heat transfer in a heat-generating fin with radiation and convection with temperature-dependent heat transfer coefficient.

Further, there are many studies available on the effect of cross-diffusion on onset of double-diffusive convection in a porous medium. Thermal convection in a binary fluid driven by the Soret and Dufour effects has been investigated by Knobloch [10]. Hurle and Jakeman [11] performed a theoretical study of Soret driven thermosolutal convection in a binary fluid mixture. Linear and nonlinear analyses of double diffusive convection in a fluid saturated porous layer with cross-diffusion effects has been carried out by Malashetty and Biradar [12]. Rudraiah and Malashetty [13] carried out a study on double diffusive convection in a porous medium in the presence of Soret and Dufour effects, while Gaikwad et. al. [14]-[16] performed a linear and nonlinear double diffusive convection in a fluid-saturated anisotropic porous layer with cross-diffusion and obtained the effect of cross diffusion coefficients. Bhadauria, Hashim et al. [17] investigated the double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect and internal heat source. Rudraiah, Siddheshwar [18] did a weak nonlinear stability analysis of double diffusive convection with cross diffusion in a fluid saturated porous medium and obtained some very interesting results.

Convection in binary fluids is a complex process. The presence of concentration currents as well as thermal currents leads to linear and nonlinear behavior. In a binary fluid, the density depends on both temperature and solute concentration. This leads to a competition between heat diffusion and solute diffusion, and consequently oscillatory motions may occur. The oscillatory convective instability in binary fluid mixtures is well understood by Platten and Legros [19]. Taslim and Narusawa [20] investigated binary fluid composition and double diffusive convection in a porous medium. Further, the studies of double diffusive convection in porous media plays very significant roles in many areas such as in petroleum industry, solidification of binary mixture, migration of solutes in water saturated soils. Other examples include; geophysics system, crystal growth, electrochemistry, the migration of moisture through air contained in fibrous insulation, Earth's oceans, magma chambers etc. The studies on double diffusive convection in a porous media has been presented in details by Ingham and Pop [21], Nield and Bejan [22] and vafai [23]-[24] and Vadasz [25] in their books. Further, it was performed by many other researchers, namely;

Poulikakos [26], Travison and Bejan [27], Momou [28] etc. The very first study on double diffusive convection in porous media was mainly concerned with linear stability analysis, and was performed by Nield [29].

It is well known that the Darcy's law is not valid for non-Newtonian fluid flows in porous media. Swamy et al [30] studied the onset of Darcy-Brinkman convection in a binary viscoelastic fluid saturated porous layer, where the modified Darcy-Brinkman-Oldroyd model has been developed. However, published works on thermal convection of viscoelastic fluids in porous media are fairly limited. Rudraiah et al. [31] have studied the thermal stability of a viscoelastic fluid saturated sparsely packed porous layer. Kim et al. [32] studied the thermal instability of viscoelastic fluids in a porous medium by performing linear and nonlinear analyses. Yoon et al. [33] analyzed the onset of thermal convection in a horizontal porous layer saturated with a viscoelastic liquid using a linear theory. Zhang et al. [34] carried out linear and nonlinear analyses of thermal convection for Oldroyd-B fluids in porous media, heated from below. Gaikwad and Kouser [35] investigated the onset of Darcy-Brinkman convection in a binary viscoelastic fluid saturated porous layer with internal heat source. Gaikwad and Dhanraj [36] studied Soret effect on Darcy-Brinkman convection in a binary viscoelastic fluid-saturated porous layer and studied the cross diffusion effects on convective instability. Stability analysis of Soret-driven double diffusive convection of a Maxwell fluid in a porous medium has been investigated by Wang and Tan [37]. Narayana et al. [38] performed linear and nonlinear stability analysis of binary Maxwell fluid convection in a porous medium with Soret and Dufour effects. Rudraiah et al. [39] have studied the stability of a viscoelastic fluid saturated sparsely packed porous layer. Malashetty et al. [40] have investigated the onset of convection in a binary viscoelastic fluid saturated porous layer. Kumar and Bhadauria [41] performed stability analysis to study thermal instability in a rotating anisotropic porous layer saturated by a viscoelastic fluid.

Malashetty et al. [42] did an analytical study of linear and nonlinear double diffusive convection with soret effect in couple stress liquids. More recently, Gaikwad and Kamble [43] have studied theoretically, the cross diffusion effects on convective instability in porous media and Gaikwad et al. [44] have performed a study on double diffusive convection in a binary viscoelastic fluid saturated porous layer with Soret effect and internal heat source. Therefore, in the present paper, we have carried out linear and nonlinear stability analyses and studied the effect of internal heat and Soret parameter on Darcy-Brinkman convection in a binary viscoelastic fluid saturated porous layer.

2. MATHEMATICAL FORMULATION

Consider a viscoelastic fluid saturated porous layer, confined between two infinitely extended horizontal planes at $z=0$ and $z=d$ heated from below and cooled from above. An internal heat source term has been included in the energy equation. A cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z-axis as vertical upward. An adverse temperature gradient is applied across the porous layer and the lower and upper planes are kept at temperature $T_0+\Delta T$, and T_0 with concentration $S_0+\Delta S$ and S_0 respectively. The governing equations are as given

$$\nabla . q = 0 \tag{1a}$$

$$\left(1+\overline{\lambda}_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial q}{\partial t}+\nabla p-\rho g\right)=\left(1+\overline{\lambda}_2\frac{\partial}{\partial t}\right)\left(\mu_c\nabla^2 q-\frac{\mu}{\kappa}q\right) \tag{1b}$$

$$\gamma\frac{\partial T}{\partial t}+\left(q\nabla\right)T=K_{11}\nabla^2 T+Q\left(T-T_0\right) \tag{1c}$$

$$\varepsilon\frac{\partial S}{\partial t}+\left(q\nabla\right)S=K_{22}\nabla^2 S+K_{21}\nabla^2 T \tag{1d}$$

$$\rho=\rho_0\left[1-\beta_T\left(T-T_0\right)+\beta_S\left(S-S_0\right)\right] \tag{1e}$$

where the physical variables have their usual meanings as given in the nomenclature. The externally imposed the thermal and solutal boundary conditions are given by

$$T=T_0+\Delta T;\ \ at\ \ z=0\ \ and\ \ T=T_0;\ \ at\ z=d;$$
$$S=S_0+\Delta S;\ \ at\ z=0\ \ and\ \ S=S_0;\ \ at\ z=d; \tag{2}$$

2.1. BASIC SOLUTION

At this state the velocity, pressure, temperature and density profiles are given by

$$q_b=0, p=p_b(z), T=T_b(z), S=S_b(z), \rho=\rho_b(z). \tag{3}$$

Substituting Eq. (3) in Eq. (1a-1e), we get the following relations:

$$\frac{dp_b}{dz}=-\rho_b g, \tag{4}$$

$$K_{11}\frac{d^2T_b}{dz^2}+Q(T_b-T_0)=0, \quad (5)$$

$$\frac{d^2S_b}{dz^2}=0, \quad (6)$$

$$\rho_b=\rho_0\left[1-\beta_T(T_b-T_0)+\beta_S(S_b-T_0)\right]. \quad (7)$$

The solution of Eq. (5), subject to the boundary condition (2),is given by

$$T_b=T_0+\Delta T\frac{sin\sqrt{R_i}\left(1-\frac{z}{d}\right)}{sin\sqrt{R_i}}. \quad (8)$$

The solution of Eq. (5), subject to the boundary condition (2),

$$S_b=S_0+\Delta S\left(1-\frac{z}{d}\right) \quad (9)$$

Now, we superimpose finite amplitude perturbations on the basic state in the form:

$$q=q_b+q',\ T=T_b+T',\ p=p_b+p',\ S=S_b+S',\ \rho=\rho_b+\rho', \quad (10)$$

We get the following set of equations:

$$\nabla.q'=0$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial q'}{\partial t}+\nabla p'-\rho(\beta_T T'-\beta_S S')g\right)=\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\mu_c\nabla^2q'-\frac{\mu}{\kappa}q'\right)$$

$$\gamma\frac{\partial T'}{\partial t}+(q'.\nabla)T'+w'\frac{\partial T_b'}{\partial z}=K_{11}\nabla^2T'+QT'$$

$$\varepsilon\frac{\partial S'}{\partial t}+(q'.\nabla)S-w'\frac{\Delta S}{d}=K_{22}\nabla^2S'+K_{21}\nabla^2T'$$

$$\rho'=-\rho_0\left[\beta_T T'+\beta_S S'\right]$$

Infinitesimal perturbation was applied to the basic state of the system and then the pressure term was eliminating by taking curl twice of Eq. (1b). The above resulting equations are non-dimensionalized using the following transformations,

$$(x',y',z')=(x^*,y^*,z^*)d,\ t'=t^*(\frac{\gamma d^2}{K_{11}}),\ q=\frac{K_{11}}{d}q^*,\ (u,v,w)=(u^*,v^*,w^*)(\frac{K_{11}}{d}),\ T'=(\Delta T)T^*,\ S'=(\Delta S)S^* \quad (11)$$

T_b, S_b in dimensionless forms are given as

$$T_b=(1-z),\ S_b=(1-z) \quad (12)$$

The non dimensionalized equations (on dropping the asterisks for simplicity) are

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2w-Ra_T\nabla_1^2T+Ra_S\nabla_1^2S\right)-\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(D_a\nabla^4w-\nabla^2w\right)=0 \quad (13)$$

$$\left[\frac{\partial}{\partial t}-\nabla^2-R_i+q.\nabla\right]T-w=0 \quad (14)$$

$$\left[\varepsilon_n\frac{\partial}{\partial t}-\nabla^2\frac{1}{L_e}\right]S+(q.\nabla)S-S_r\nabla^2T-w=0 \quad (15)$$

where $Pr_D=\frac{\varepsilon\gamma\nu d^2}{K_{11}K}$ is Darcy-Prandtl number, $Ra_T=\frac{\beta_T g\Delta SKd}{\nu K_{11}}$ is the thermal Rayleigh number, $Ra_S=\frac{\beta_S g\Delta SKd}{\nu K_{11}}$ is the solute Rayleigh number, $R_i=\frac{Qd^2}{K_{11}}$ is the internal Rayleigh parameter, $\lambda_1=(\frac{K_{11}}{\gamma d^2})\overline{\lambda_1}$ is relaxation parameter , $\lambda_2=(\frac{K_{11}}{\gamma d^2})\overline{\lambda_2}$ is retardation parameter, $L_e=\frac{K_{11}}{K_{22}}$ is Lewis number, $S_r=\frac{K_{21}\Delta T}{K_{11}\Delta S}$ the soret parameter, $\varepsilon_n=\frac{\varepsilon}{\gamma}$ normalized porosity. The above system will be solved by considering stress free and isothermal boundary conditions as given below:

$$w=\frac{\partial^2w}{\partial z^2}=T=S=0\ \text{ at }\ z=0,z=1. \quad (16)$$

3. LINEAR STABILITY ANALYSIS

In order to do the linear stability analysis of the system, Eq. (13)-(15) subject to the boundary condition given in Eq.(16), we use time dependent periodic disturbance in horizontal plane as

$$(w,T,S)=(W,\Theta,\phi)exp\left(i\left(lx+my\right)+\sigma t\right), \qquad (17)$$

Where α are horizontal wave number and $\sigma=\sigma_r+i\sigma_j$ is growth rate. Substituting eq. (17) into the lineralized eq. (13)-(15). We obtain

$$(1+\lambda_1\sigma)\left(\frac{\sigma}{\Pr_D}\left(D^2-a^2\right)W+a^2Ra_T\Theta-a^2Ra_S\phi\right)-(1+\lambda_2\sigma)\left(D^2-a^2\right)\left[D_a\left(D^2-a^2\right)^2-1\right]W=0 \qquad (18)$$

$$\left[\sigma-(\mathrm{D}^2-\mathrm{a}^2)-\mathrm{Ri}\right]\Theta-\mathrm{W}=0 \qquad (19)$$

$$\left[\varepsilon_n\sigma+\frac{D^2-a^2}{L_e}\right]\phi-W-(D^2-a^2)S_r\Theta=0. \qquad (20)$$

Where $a^2=l^2+m^2$. The boundary conditions (16) are now

$$W=\frac{\partial^2 W}{\partial z^2}=\Theta=\phi=0 \;\; on \;\; z=0, z=1. \qquad (21)$$

We assume the solution W,Θ,ϕ as

$\left(W,\Theta,\phi\right)=\left(W_0,\Theta_0,\phi_0\right)\mathrm{Sin}\,n\pi z$ (n=1,2,3........),

The most unstable mode corresponds to n = 1 (fundamental mode). Therefore, substituting Eq. (21) with n = 1 into Eq. (18)-(20), we obtain a matrix form $Ax=0$ as

$$\begin{pmatrix} \left(\frac{\sigma}{\Pr_D}+\Lambda\left(D_a\delta^2+1\right)\right)\delta^2 & -a^2Ra_T & a^2Ra_S \\ -1 & \left(\sigma+\delta^2-R_i\right) & 0 \\ -1 & S_r\delta^2 & \varepsilon_n\sigma+\frac{\delta^2}{Le} \end{pmatrix}\begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix}=0. \qquad (22)$$

The thermal Rayleigh number can be expressed as

$$Ra_T=\frac{\delta^2}{a^2}\left(\frac{\sigma}{Pr_D}+\wedge(D_a\delta^2+1)\right)(\sigma+\delta^2-R_i)+\frac{\sigma-R_i+\delta^2(1-S_r)}{\varepsilon_n\sigma+\frac{\delta^2}{L_e}}Ra_S$$

(23)

Where $\delta^2=\pi^2+a^2$, $\Lambda=\frac{1+\lambda_2\sigma}{1+\lambda_1\sigma}$. The growth rate σ is in general a complex quantity such that $\sigma=\sigma_r+i\,\sigma_i$. The system with $\sigma_r<0$ is always stable, while for $\sigma_r>0$ it will become unstable. For neutral stability state $\sigma_r=0$

3.1. STATIONARY STATE

We now set $\sigma=0$ at the margin of stability. The expressed for the thermal Rayleigh number of the system for a stationary mode of convection is as given below:

$$Ra_T^{st}=\frac{\pi^2+a^2}{a^2}(1+D_a\delta^2)(\delta^2-R_i)+\frac{\left(\delta^2(1-S_r)-R_i\right)L_e}{\delta^2}Ra_S,$$

(24)

It is important to note that the critical wave number $a=a_c^{St}$, where $a_c^{St}=\sqrt{S}$ satisfied the following equation

$$2D_aS^3+(3D_a\pi^2+1)S^2-\pi^4(D_a\pi^2+1)=0 \qquad (25)$$

In the absence of Soret effect i.e. $S_r=0$ Eq. (24) becomes

$$Ra_T^{st} = \frac{\pi^2 + a^2}{a^2}(1 + D_a\delta^2)(\delta^2 - R_i) + \frac{(\delta^2 - R_i)L_e}{\delta^2} Ra_S. \tag{26}$$

For the system without internal-heating, i.e., $R_i = 0$ we get

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2}(1 + D_a\delta^2) + L_e Ra_S \tag{27}$$

This is exactly the same as obtained by Swamy et al. [30]. When $D_a \to 0$

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2} + L_e Ra_S \tag{28}$$

In case of single component fluid, the Solutal Rayleigh number is zero i.e. $Ra_S = 0$, we have

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2} \tag{29}$$

Which is the classical result obtained by Horton and Rogers [2] and Lapwood [3] for single component fluid in porous layer.

3.2. OSCILLATORY STATE

We set $\sigma = i\sigma_i$ in Eq. (23) and clear the complex quantities from the denominator, to obtain

$$Ra_T^{osc} = \Delta_1 + i\sigma_i\Delta_2. \tag{30}$$

Where

$$\Delta_1 = \frac{\delta^2}{a^2}\left[\left(\delta^2 - R_i\right)\left(D_a\delta^2 + 1\right)\left(\frac{1 + \lambda_1\lambda_2\sigma^2}{1 + \lambda_1{}^2\sigma^2}\right) - \sigma^2\left(\frac{1}{\mathrm{Pr}_D} + \frac{\left(D_a\delta^2 + 1\right)\left(\lambda_2 - \lambda_1\right)}{1 + \lambda_1{}^2\sigma^2}\right)\right] + \frac{\varepsilon_n\sigma^2 + \delta^4 L_e{}^{-1}\left(1 - S_r\right)\delta^2 L_e{}^{-1}R_i}{\left(\delta^2 L_e{}^{-1}\right)^2 + \varepsilon_n{}^2\sigma^2} Ra_S$$

$$\Delta_2 = \frac{\delta^2}{a^2}\left[\left(\delta^2 - R_i\right)\left(\frac{1}{\mathrm{Pr}_D} + \frac{\left(D_a\delta^2 + 1\right)\left(\lambda_2 - \lambda_1\right)}{1 + \lambda_1{}^2\sigma^2}\right) + \left(D_a\delta^2 + 1\right)\left(\frac{1 + \lambda_1\lambda_2\sigma^2}{1 + \lambda_1{}^2\sigma^2}\right)\right] + \frac{\delta^2 L_e{}^{-1} - \varepsilon_n\left(\delta^2\left(1 - S_r\right) - R_i\right)}{\left(\delta^2 L_e{}^{-1}\right)^2 + \varepsilon_n{}^2\sigma^2} Ra_S.$$

For oscillatory mode $\Delta_2 = 0$ and $\sigma_i \neq 0$, which is not given for brevity. The thermal Rayleigh number for oscillatory mode is given as:

$$Ra_T^{osc} = \Delta_1 \tag{31}$$

4. NONLINEAR STABILITY ANALYSIS

In this section, we study the nonlinear stability analysis using minimal truncated Fourier series. For simplicity, we confine ourself only to two dimensional rolls, so that all the physical quantities are independent of y. Introducing the stream function ψ as $u = \frac{\partial\psi}{\partial z}$, w= $-\frac{\partial\psi}{\partial x}$ and taking curl of Eq. (1 b) to eliminate pressure term we get

$$\left(1 + \lambda_1\frac{\partial}{\partial t}\right)\left(\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2\psi + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x}\right) = \left(1 + \lambda_2\frac{\partial}{\partial t}\right)\left(D_a\nabla^4\psi - \nabla^2\psi\right) \tag{32}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2 - R_i\right)T - \frac{\partial\left(\psi, T\right)}{\partial\left(x, z\right)} + \frac{\partial\psi}{\partial x} = 0 \tag{33}$$

$$\left(\varepsilon_n\frac{\partial}{\partial t} - L_e{}^{-1}\nabla^2\right)S - \frac{\partial\left(\psi, S\right)}{\partial\left(x, z\right)} + \frac{\partial\psi}{\partial x} - S_r\nabla^2 T = 0 \tag{34}$$

It is to be noted that the effect of nonlinearity is to distort the temperature concentration fields through the interaction of ψ and T , ψ and S . As a result a component of the form $Sin(2\pi z)$ will be generated. A minimal Fourier series which describes the finite amplitude convection is given by

$$\psi = A_1(t)sin(ax)sin(\pi z), \tag{35}$$

$$T = A_2(t)cos(ax)sin(\pi z) + A_3(t)sin(2\pi z), \tag{36}$$

$$S = A_4(t)cos(ax)sin(\pi z) + A_5(t)sin(2\pi z), \tag{37}$$

Where the amplitudes $A_1(t)$, $A_2(t)$, $A_3(t)$, $A_4(t)$, $A_5(t)$ are functions of time and are to be determined. Substituting above expressions in Eq. (13)-(15) and equating the like terms, the following set of nonlinear autonomous differential equations were obtained

$$\frac{dA_1}{dt} = B \tag{38}$$

$$\frac{dB}{dt} = -\frac{Pr_D}{\delta^2 \lambda_1}\left[\left(\frac{\delta^2}{Pr_D} + \lambda_2 D_a \delta^4 + \delta^2 \lambda_2\right)B + \delta^2\left(1 + D_a \delta^2\right)A_1 + aRa_T A_2 - aRa_S A_4 + a\lambda_1 Ra_T \frac{dA_2}{dt} - a\lambda_1 Ra_S \frac{dA_4}{dt}\right] \tag{39}$$

$$\frac{dA_2}{dt} = -[aA_1 + (\delta^2 - R_i)A_2 + \pi a A_1 A_3] \tag{40}$$

$$\frac{dA_3}{dt} = (R_i - 4\pi^2)A_3 + \frac{\pi a}{2}A_1 A_2 \tag{41}$$

$$\frac{dA_4}{dt} = -\frac{1}{\varepsilon_n}(aA_1 + L_e^{-1}\delta^2 A_4 + \pi a A_1 A_5 + S_r \delta^2 A_2) \tag{42}$$

$$\frac{dA_5}{dt} = -\frac{1}{\varepsilon_n}(4\pi^2 L_e^{-1} A_5 - \frac{\pi a}{2}A_1 A_4 + 4\pi^2 S_r A_3) \tag{43}$$

4.1. STEADY FINITE AMPLITUDE MOTIONS

We set $\frac{\partial}{\partial t} = 0$, the above system becomes

$$B=0 \tag{44}$$

$$\delta^2(1 + D_a \delta^2)A_1 + aRa_T A_2 - aRa_S A_4 = 0 \tag{45}$$

$$aA_1 + (\delta^2 - R_i)A_2 + \pi a A_1 A_3 = 0 \tag{46}$$

$$(R_i - 4\pi^2)A_3 + \frac{\pi a}{2}A_1 A_2 = 0 \tag{47}$$

$$aA_1 + L_e^{-1}\delta^2 A_4 + \pi a A_1 A_5 + S_r \delta^2 A_2 = 0 \tag{48}$$

$$4\pi^2 L_e^{-1} A_5 - \frac{\pi a}{2}A_1 A_4 + 4\pi^2 S_r A_3 = 0 \tag{49}$$

Numerical method was used to solve the above nonlinear differential equation to find the amplitudes. On solving for the amplitudes in terms of A_1 ,we obtain A_2, A_3, A_4, A_5 .

4.2. STEADY HEAT AND MASS TRANSPORTS

In the study of this type problem, quantification of heat and mass transport is very important.
If H and J are the rate of heat and mass transport per unit area,then

$$H = -K_{11}\langle\frac{\partial T_{total}}{\partial z}\rangle_{z=0} \tag{50}$$

$$J = -K_{21}\langle\frac{\partial T_{total}}{\partial z}\rangle_{z=0} - K_{22}\langle\frac{\partial S_{total}}{\partial z}\rangle_{z=0} \tag{51}$$

Where the angular bracket corresponds to a horizontal average and

$$T_{total} = T_0 - \Delta T\frac{z}{d} + T(x,z,t) \tag{52}$$

$$S_{total} = S_0 - \Delta S\frac{z}{d} + S(x,z,t) \tag{53}$$

Substituting Eq. (36)-(37) into Eq. (53) and using the resultant Eq. (50), (51) we get

$$H = \frac{K_{11}\Delta T}{d}(1 - 2\pi A_3) \tag{54}$$

$$J = \frac{K_{22}\Delta S}{d}\left[(1 - 2\pi A_5) + S_r L_e (1 - 2\pi A_3)\right] \tag{55}$$

The Nusselt number and Sherwood number , which denotes the rate of heat and mass transports respectively, are defined by

$$Nu = \frac{H}{\frac{K_{11}\Delta T}{d}} = (1 - 2\pi A_3) \tag{56}$$

$$Sh = \frac{J}{\frac{K_{22}\Delta S}{d}} = (1 - 2\pi A_5) + S_r L_e (1 - 2\pi A_3) \tag{57}$$

Using the expressions Eq.(54)-(55), and substituting A_3, A_5 into Eq. (56,57) ,finally the expressions for N_u , S_h are obtained.

5. RESULTS AND DISCUSSION

This paper investigates the combined effect of internal heating and Soret parameter on stationary and oscillatory convection in a porous medium saturated with a binary viscoelastic fluid and discusses the effects of various parameters on the onset of double diffusive convection. The expressions for the stationary and oscillatory modes of convection for different values of the parameters such as Prantdl number, relaxation parameter, retardation parameter, solute Rayleigh number, Lewis number, Soret parameter and Darcy number are computed, and the results are depicted in figures. The neutral stability curves in the (Ra_T, a) plane for various parameter values are as shown in Fig. 1 and Fig. 2. We fixed the values for the parameters as $\Pr_D = 10$, $D_a = .1$, R_i = 3 $\lambda_1 = .8$, $\lambda_2 = .1$, $Ra_S = 100$, $L_e = 2$, $S_r = .05$, and except the varying parameter.

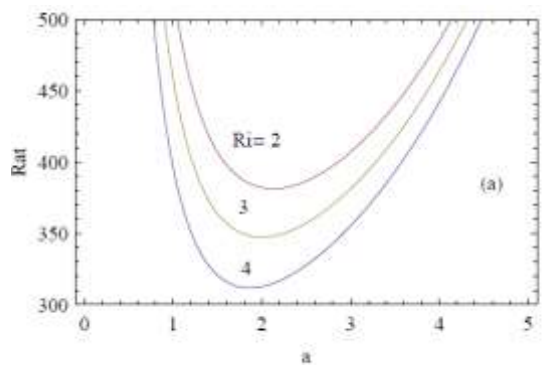

Fig. 1: Variation of Stationary rayleigh number with wave number for the different values of *Ri*

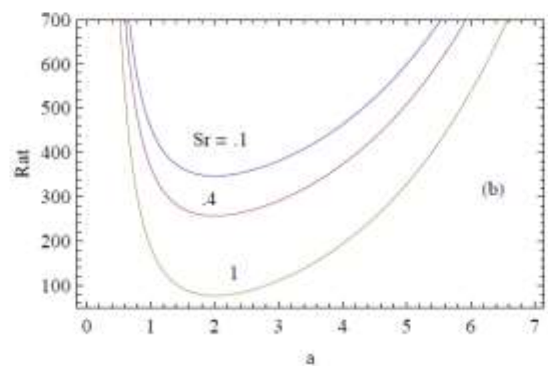

Fig. 1: Variation of Stationary rayleigh number with wave number for the different values of *Sr*

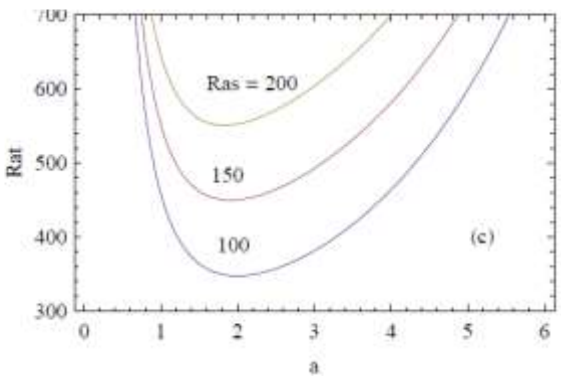

Fig. 1: Variation of Stationary rayleigh number with wave number for the different values of *Ras*

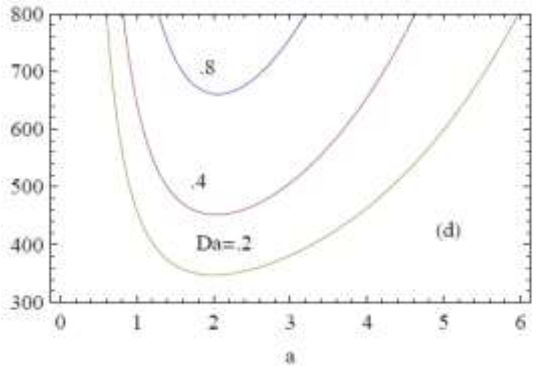

Fig. 1: Variation of Stationary rayleigh number with wave number for the different values of D_a

From Figs.1, 2(a), it is observed that increasing the value of internal heat source R_i, decreases the values of stationary and oscillatory Rayleigh number, which means that the effect of increasing the internal heat source R_i is to destabilize the system. In Figs.1, 2(b), the effect of Soret parameter (S_r) is depicted, respectively for both stationary and oscillatory convection. It is found that an increment in the value of Soret parameter decreases the value of Rayleigh numbers for both stationary and oscillatory mode of convection, thus onset of convection takes place at an early point. Figs.1, 2(c) depicts the effect of solute Rayleigh number Ra_S on the onset of convection. We find that the effect of increasing the value of Ra_S is to increase the value of Rayleigh number Ra_T thus stabilizing the system in both stationary and oscillatory modes. Further, Figs.1, 2(d) show that the effect of increasing the Darcy number, D_a is to increase the value of Rayleigh number Ra_T, thus stabilizing the system that is the onset of convection will take place at a later point. However, the effect of increasing the Lewis number L_e is found to increase the value of Rayleigh number for stationary mode and decrease the value for oscillatory modes, thus to stabilize the stationary mode of convection and destabilize the oscillatory convection [Figs.1, 2(e)].

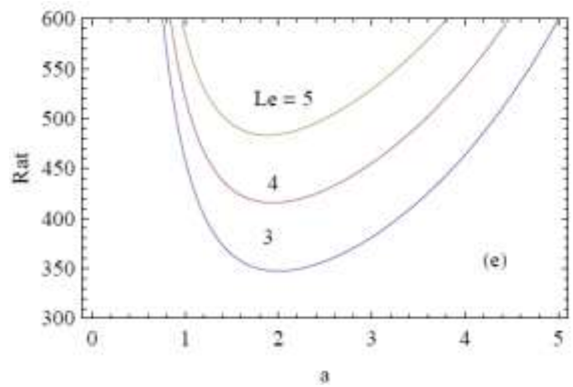

Fig. 1: Variation of Stationary rayleigh number with wave number for the different values of *Le*

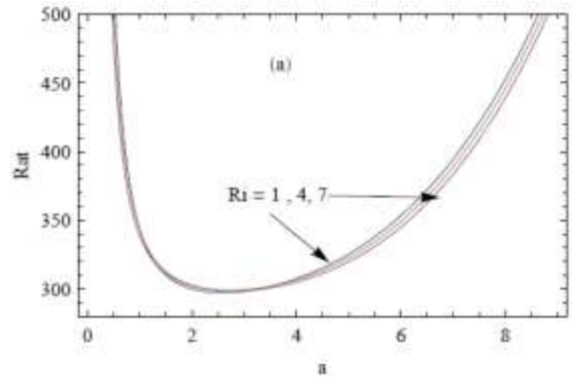

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of *Ri*

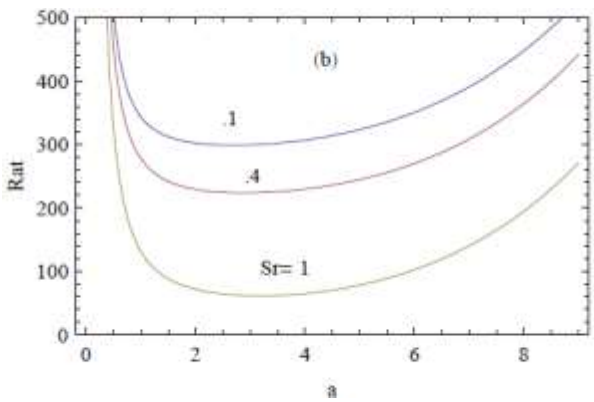

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of *Sr*

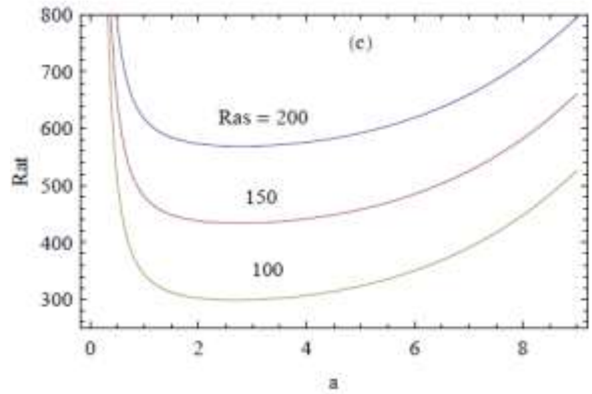

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of *Ras*

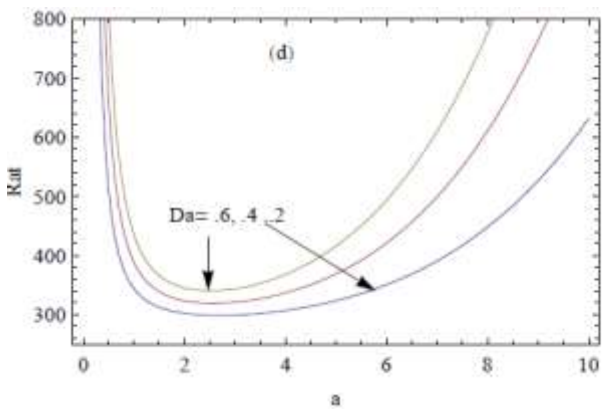

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of *Da*

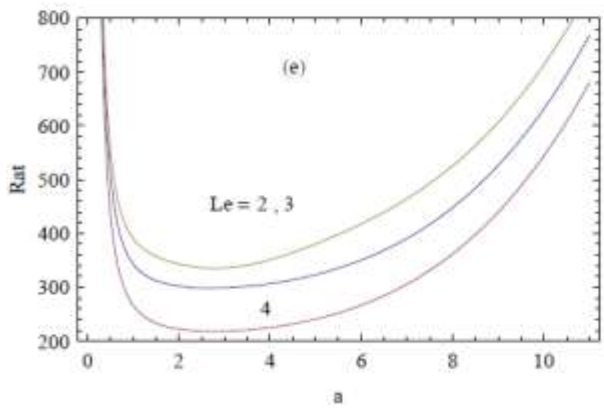

Fig. 2 : Variation of Oscillatory rayleigh number with wave number for the different values of *Le*

Also, from Figs. 2(f, g), we find that the oscillatory Rayleigh number decreases on increasing the value of the relaxation parameter λ_1 and Prandtl number Pr_D, indicating that the effect of relaxation parameter and the Prandtl number is to destabilize the system. Thus, the oscillatory convection takes place at an early point. However, from Fig.2 (h), the effect of retardation parameter λ_2 is found to stabilize the system, thus opposite to that due to λ_1.

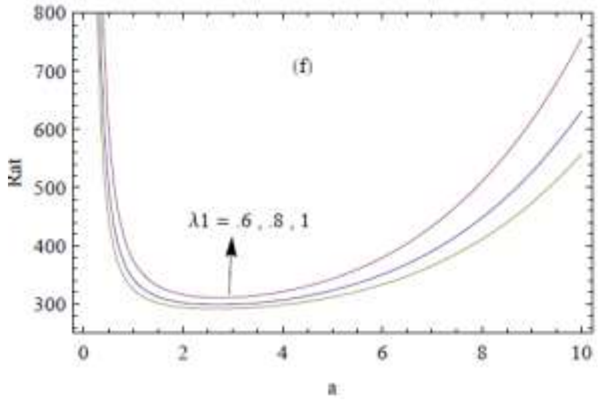

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of λ_1

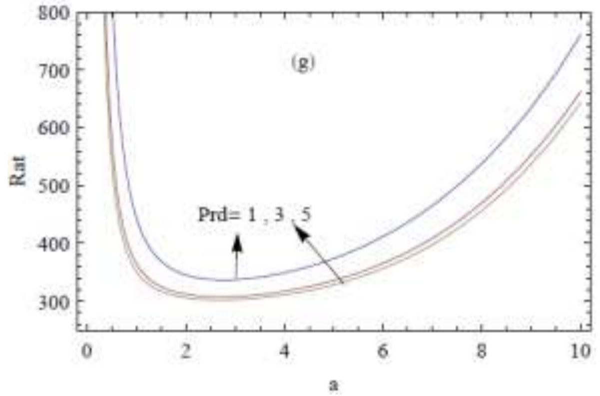

Fig. 2: Variation of Oscillatory rayleigh number with wave number for the different values of *Prd*

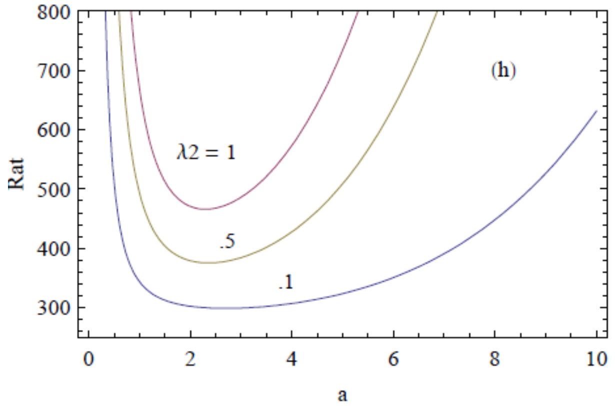

Fig. 2 : Variation of Oscillatory rayleigh number with wave number for the different values of λ_2

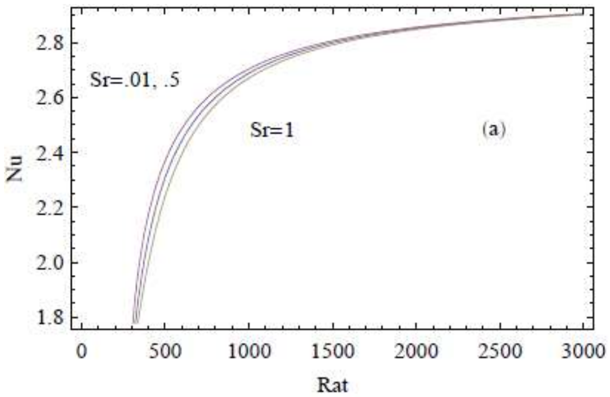

Fig. 3: Variation of Nusselt number with rayleigh number for the different values of Sr

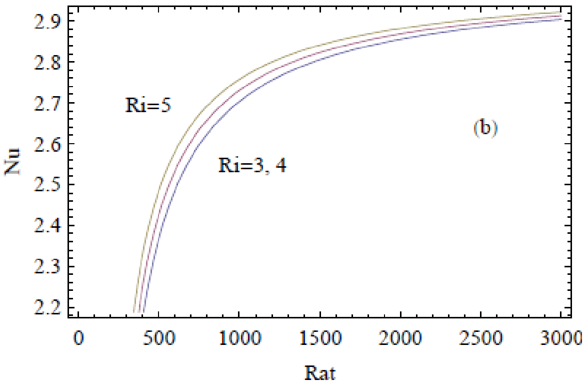

Fig. 3: Variation of Nusselt number with rayleigh number for the different values of Ri

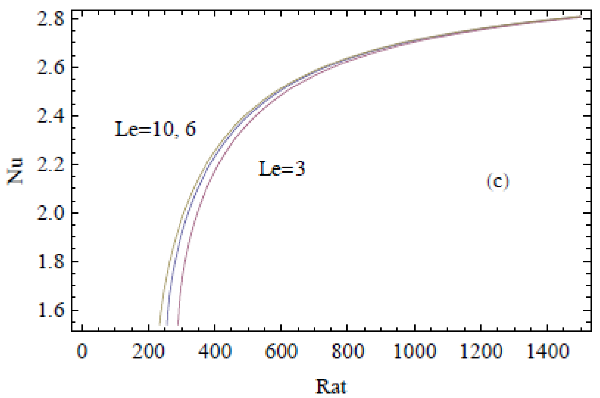

Fig. 3: Variation of Nussult number with rayleigh number for the different values of Le

Now, we fix the values of the parameters as Ra_S = 100, L_e = 2, D_a =.1, S_r = .05 and R_i = 3 to compute the heat and mass transports across the porous medium. The results have been obtained for steady state motion, in terms of the Nussult and Sherwood numbers and depicted in the Figs.3, 4 respectively. It is found that the steady state values of N_u and S_h approach 3 as Ra_T increases. Further, it is found from Figs.3,4(a) that the value of N_u decreases, while that of S_h increases on increasing the values of Soret parameter S_r This shows that the effect of Soret parameter is to decrease the heat transport, thus stabilizing the system and increase the mass transport in the system. In Figs. 3(b, c) and 4(b, c), it is found that heat and mass transports increase on increasing R_i and L_e , thus destabilizing the system. However, Ra_S has a stabilizing effect on the system as heat and mass transport decrease on increasing the value of Ra_S [Fig.3, 4(d)].

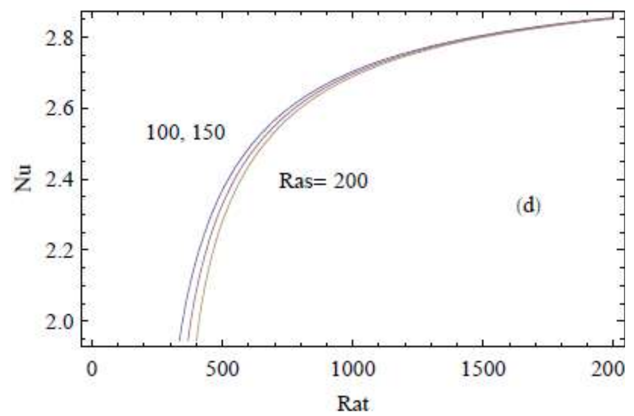

Fig. 3: Variation of Nussult number with rayleigh number for the different values of Ras

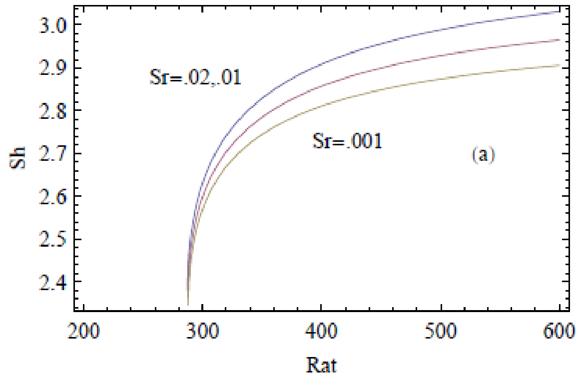

Fig. 4: Variation of Sherewood number with rayleigh number for the different values of *Sr*

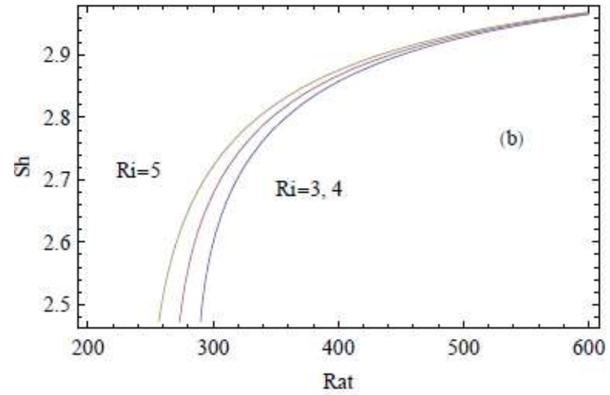

Fig. 4: Variation of Sherewood number with rayleigh number for the different values of Ri

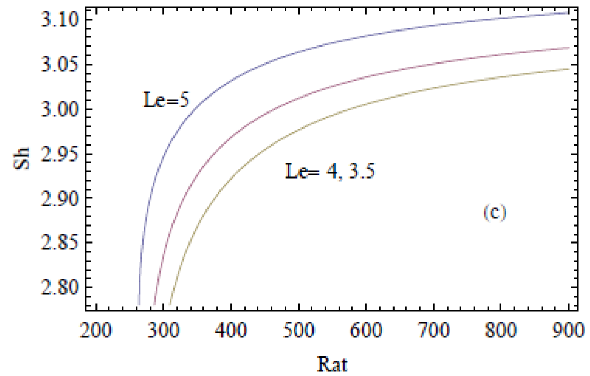

Fig. 4: Variation of Sherewood number with rayleigh number for the different values of Le

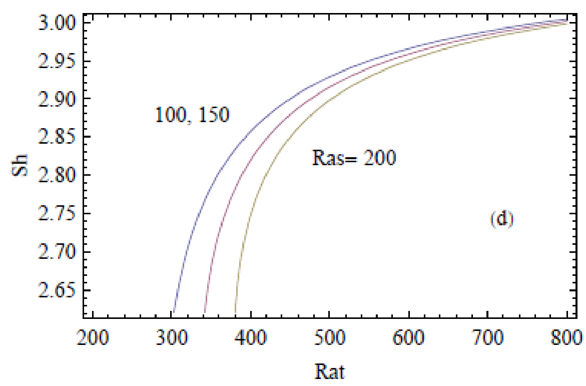

Fig. 4: Variation of Sherewood number with rayleigh number for the different values of Ras

6. CONCLUSIONS

Effects of Soret parameter and internal heat source on double diffusive convection in a binary viscoelastic fluid saturated porous layer, heated and salted from below, is investigated analytically using linear and nonlinear stability analysis. Following conclusions are drawn:

1) The Internal heat source R_i and Soret parameter S_r have destabilizing effect on the system in both stationary and oscillatory modes of convection.
2) The Darcy number D_a and Solute Rayleigh number Ra_S have stabilizing effect on the both stationary and oscillatory convection.
3) The Lewis number L_e has stabilizing effect on stationary mode of convection while destabilizing effect on oscillatory mode of convection.
4) Relaxation parameter λ_1 and Prandtl number Pr_D have destabilizing effect, while retardation parameter λ_2 has stabilizing effect on the oscillatory convection.
5) Increments in Lewis number L_e and internal Rayleigh number R_i increase, while in Ra_S decrease heat and mass transports in the system.
6) Effect of Soret parameter S_r is to decrease the heat transfer and increase the mass transfer in the system.

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