

# $(g\#, s)$ -continuous functions in topology

Nagendran R, Latha R, Karpagam S  
Assistant Professor

Sree Sastha Institute Of Engineering And Technology, Chennai, India

**Abstract**—In this paper, we introduce  $(g\#, s)$ -continuous functions between topological spaces, study some of its basic properties and discuss its relationships with other topological functions.

**IndexTerms**—  $g\#$ -closed set, regular open set,  $g\#$ -T1/2 space,  $\pi g$ -T1/2 space,  $(g\#, s)$ -continuous function.

## I. INTRODUCTION

It is well known that the concept of closedness is fundamental with respect to the investigation of general topological spaces. Levine [26] initiated the study of generalized closed sets. The concept of  $g\#$ -closed sets was introduced by Veerakumar [46]. Recently, this notion is further studied by Ravi et al [40]. Initiation of contra-continuity was due to Dontchev [10]. Many different forms of contra-continuity functions have been introduced over the years by various authors [5,11,14,15,17,19,22,37,40].

In this paper, new generalizations of contra-continuity by using  $g\#$ -closed sets called  $(g\#, s)$ -continuity are presented. Characterizations and properties of  $(g\#, s)$ -continuous functions are discussed in detail. Finally, we obtain many important results in topological spaces

## II. PRELIMINARIES

In this paper, spaces  $X$  and  $Y$  mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset  $A$  of a space  $X$ ,  $cl(A)$  and  $int(A)$  represent the closure of  $A$  and interior of  $A$  respectively.

A subset  $A$  of a space  $X$  is said to be regular open (resp. regular closed) if  $A = int(cl(A))$  (resp.  $A = cl(int(A))$ ) [43]. The  $\delta$ -interior [45] of a subset  $A$  of  $X$  is the union of all regular open sets of  $X$  contained in  $A$  and it is denoted by  $\delta-int(A)$ . A subset  $A$  is called  $\delta$ -open [45] if  $A = \delta-int(A)$ . The complement of  $\delta$ -open set is called  $\delta$ -closed. The  $\delta$ -closure of a set  $A$  in a space  $(X, \tau)$  is defined by  $\delta-cl(A) = \{ x \in X: A \cap int(cl(U)) \neq \emptyset, U \in \tau \text{ and } x \in U \}$  and it is denoted by  $\delta-cl(A)$ .

The finite union of regular open sets is said to be  $\pi$ -open [48]. The complement of  $\pi$ -open set is said to be  $\pi$ -closed. A subset  $A$  is said to be semi-open [25] (resp.  $\alpha$ -open [31], preopen [30],  $\beta$ -open [1] or semi-preopen [2]) if  $A \subset cl(int(A))$  (resp.  $A \subset int(cl(int(A)))$ ,  $A \subset int(cl(A))$ ,  $A \subset cl(int(cl(A)))$ ). The complement of semi-open (resp.  $\alpha$ -open, preopen,  $\beta$ -open) is said to be semi-closed (resp.  $\alpha$ -closed, preclosed,  $\beta$ -closed). The union (resp. intersection) of all  $\alpha$ -open (resp.  $\alpha$ -closed) sets, each contained in (containing) a set  $S$  in a topological space  $X$  is called  $\alpha$ -interior (resp.  $\alpha$ -closure) of  $S$  and it is denoted by  $\alpha cl(S)$  (resp.  $\alpha int(S)$ ). The union (resp. intersection) of all semi-open (resp. semi-closed) sets, each contained in (containing) a set  $S$  in a topological space  $X$  is called semi-interior (resp. semi-closure) of  $S$  and it is denoted by  $scl(S)$  (resp.  $sint(S)$ ). The union (resp. intersection) of all preopen (resp. preclosed) sets, each contained in (containing) a set  $S$  in a topological space  $X$  is called preinterior (resp. preclosure) of  $S$  and it is denoted by  $pcl(S)$  (resp.  $pint(S)$ ).

A subset  $A$  of a space  $X$  is said to be  $\alpha$ -generalized closed (briefly,  $\alpha g$ -closed)[29] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ . The complement of  $\alpha g$ -closed set is called  $\alpha g$ -open. A subset  $A$  of a space  $X$  is said to be generalized closed (briefly,  $g$ -closed) [26] (resp.  $\pi g$ -closed [13],  $g\#$ -closed [46]) if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open (resp.  $\pi$ -open,  $\alpha g$ -open) in  $X$ . The complement of  $g$ -closed (resp.  $\pi g$ -closed,  $g\#$ -closed) is said to be  $g$ -open (resp.  $\pi g$ -open,  $g\#$ -open,  $rg$ -open). The union (resp. intersection) of all  $g\#$ -open (resp.  $g\#$ -closed) sets, each contained in (containing) a set  $S$  in a topological space  $X$  is called  $g\#$ -interior (resp.  $g\#$ -closure) of  $S$  and it is denoted by  $g\#-int(S)$  (resp.  $g\#-cl(S)$ ).

A point  $x \in X$  is said to be a  $\theta$ -semi-cluster point [23] of a subset  $A$  of  $X$  if  $cl(U) \cap A \neq \emptyset$  for every semi-open set  $U$  containing  $x$ . The set of all  $\theta$ -semi-cluster points of  $A$  is called the  $\theta$ -semi-closure of  $A$  and is denoted by  $\theta-s-cl(A)$ . A subset  $A$  is called  $\theta$ -semi-closed [23] if  $A = \theta-s-cl(A)$ . The complement of a  $\theta$ -semi-closed set is called  $\theta$ -semi-open.

The family of all  $\delta$ -open ( resp.  $g^\#$ -open,  $g^\#$ -closed,  $\pi g$ -open,  $\pi g$ -closed, regular open, regular closed, semi-open, closed) sets of  $X$  containing a point  $x \in X$  is denoted by  $\delta O(X, x)$  (resp.  $G^\#O(X, x)$ ,  $G^\#C(X, x)$ ,  $\pi GO(X, x)$ ,  $\pi GC(X, x)$ ,  $RO(X, x)$ ,  $RC(X, x)$ ,  $SO(X, x)$ ,  $C(X, x)$ ). The family of all  $\delta$ -open (resp.  $g^\#$ -open,  $g^\#$ -closed,  $\pi g$ -open,  $\pi g$ -closed, semi-open,  $\beta$ -open, preopen, regular open, regular closed) sets of  $X$  is denoted by  $\delta O(X)$  (resp.  $G^\#O(X)$ ,  $G^\#C(X)$ ,  $\pi GO(X)$ ,  $\pi GC(X)$ ,  $SO(X)$ ,  $\beta O(X)$ ,  $PO(X)$ ,  $RO(X)$ ,  $RC(X)$ ).

**Definition 1:** A space  $X$  is said to be

- (1)  $s$ -Urysohn [3] if for each pair of distinct points  $x$  and  $y$  in  $X$ , there exist  $U \in SO(X, x)$  and  $V \in SO(X, y)$  such that  $cl(U) \cap cl(V) = \phi$ .
- (2) weakly Hausdorff [41] if each element of  $X$  is an intersection of regular closed sets.

**Definition 2** [20]: Let  $B$  be a subset of a space  $X$ . The set  $\bigcap \{ A \in RO(X) : B \subset A \}$  is called the  $r$ -kernel of  $B$  and is denoted by  $r\text{-ker}(B)$ .

**Proposition 3** [20]: *The following properties hold for subsets  $A, B$  of a space  $X$ :*

- (1)  $x \in r\text{-ker}(A)$  if and only if  $A \cap K \neq \phi$  for any regular closed set  $K$  containing  $x$ .
- (2)  $A \subset r\text{-ker}(A)$  and  $A = r\text{-ker}(A)$  if  $A$  is regular open in  $X$ .
- (3)  $A \subset B$ , then  $r\text{-ker}(A) \subset r\text{-ker}(B)$ .

**Lemma 4** [28]: If  $V$  is an open set, then  $scl(V) = int(cl(V))$ . The subset  $\{(x, f(x)) : x \in X\} \subset X \times Y$  is called the graph of a function  $f : X \rightarrow Y$  and is denoted by  $G(f)$ .

hereverTimesisspecified,TimesRomanorTimesNewRomanmaybeused.Ifneitherisavailableonyourwordprocessor,pleaseuseth efontclosestinappearancetoTimes.Avoidusingbit-mappedfonts.TrueType1orOpenTypefontsare required.Pleaseembed all fonts, in particular symbolfonts,aswell,formath,etc.

### III. CHARACTERIZATIONS OF $G^\#$ -OPEN SETS

**Lemma 5:** For any subset  $K$  of a topological space  $X$ ,  $X \setminus g^\#-cl(K) = g^\#-int(X \setminus K)$ .

**Lemma 6:** *If a subset  $A$  is  $g^\#$ -closed in a space  $X$ , then  $A = g^\#-cl(A)$ .*

**Theorem 7:** If  $A$  and  $B$  are  $g^\#$ -closed sets, then  $A \cup B$  is also a  $g^\#$ -closed set.

**Proof** It follows from the fact that  $cl(A \cup B) = cl(A) \cup cl(B)$ .

**Theorem 8:** If  $A$  and  $B$  are  $g^\#$ -open sets, then  $A \cap B$  is also a  $g^\#$ -open set.

**Theorem 9:** A set  $A$  is  $g^\#$ -open in  $(X, \tau)$  if and only if  $F \subseteq int(A)$  whenever  $F$  is  $\alpha g$ -closed in  $X$  and  $F \subseteq A$ .

**Proof** Assume that  $A$  is  $g^\#$ -open,  $F \subseteq A$  and  $F$  is  $\alpha g$ -closed. Then  $X \setminus F$  is  $\alpha g$ -open and  $X \setminus A \subseteq X \setminus F$ . Since  $X \setminus A$  is  $g^\#$ -closed,  $cl(X \setminus A) \subseteq X \setminus F$ . It implies that  $X \setminus int(A) \subseteq X \setminus F$  and hence  $F \subseteq int(A)$ .

Conversely, put  $X \setminus A = B$ . Suppose  $B \subseteq U$  where  $U$  is  $\alpha g$ -open. Now if  $X \setminus A \subseteq U$ , then  $F = X \setminus U \subseteq A$  and  $F$  is  $\alpha g$ -closed. It implies that  $F \subseteq int(A)$  and hence  $X \setminus int(A) \subseteq X \setminus F = U$ . Therefore  $X \setminus int(X \setminus B) \subseteq U$  and consequently  $cl(B) \subseteq U$ . Hence  $B$  is  $g^\#$ -closed and therefore  $A$  is  $g^\#$ -open.

**Theorem 10:** Suppose that  $A$  is  $g^\#$ -open in  $X$  and that  $B$  is  $g^\#$ -open in  $Y$ . Then  $A \times B$  is  $g^\#$ -open in  $X \times Y$ .

**Proof** Suppose that  $F$  is closed and hence  $\alpha g$ -closed in  $X \times Y$  and that  $F \subseteq A \times B$ . By the previous theorem, it suffices to show that  $F \subseteq int(A \times B)$ .

Let  $(x, y) \in F$ . Then, for each  $(x, y) \in F$ ,  $cl(\{x\}) \times cl(\{y\}) = cl(\{x\} \times \{y\}) = cl(\{x, y\}) \subset cl(F) = F \subset A \times B$ . Two closed sets  $cl(\{x\})$  and  $cl(\{y\})$  are contained in  $A$  and  $B$  respectively. It follows from the assumption that  $cl(\{x\}) \subseteq int(A)$  and that  $cl(\{y\}) \subseteq int(B)$ . Thus  $\{x, y\} \in cl(\{x\}) \times cl(\{y\}) \subseteq int(A) \times int(B) \subseteq int(A \times B)$ . It means that, for each  $(x, y) \in F$ ,  $(x, y) \in int(A \times B)$  and hence  $F \subseteq int(A \times B)$ . Therefore  $A \times B$  is  $g^\#$ -open in  $X \times Y$ .

**Theorem 11:** *Let  $A$  be a subset of  $Y$ . The following holds:  $g^\#-cl(X \times A) = X \times g^\#-cl(A)$ .*

**Proof** Since  $X$  and  $g^\#-cl(A)$  are  $g^\#$ -closed, the product  $X \times g^\#-cl(A)$  is a  $g^\#$ -closed set containing  $X \times g^\#-cl(A)$ . Using definition we have  $g^\#-cl(X \times A) \subset X \times g^\#-cl(A)$ .

To prove the converse containment relation:  $X \times g\#-cl(A) \subset g\#-cl(X \times A)$ , we suppose that there exists a point  $(a, b) \notin g\#-cl(X \times A)$ . Then, there exists a  $g\#$ -open set  $W$  containing  $(a, b)$  [similar to 38; Lemma 2] such that  $W \cap (X \times A) = \emptyset$  and hence  $p(W) \cap A = \emptyset$ , where  $p: X \times Y \rightarrow Y$  is the projection onto  $Y$ . Since  $p(W)$  is  $g\#$ -open containing  $b = p(a, b) \in p(W)$ ,  $b \notin g\#-cl(A)$ . Therefore, we show  $(a, b) \notin X \times g\#-cl(A)$  and hence  $X \times g\#-cl(A) \subset g\#-cl(X \times A)$ .

**Definition 12** A function  $f: X \rightarrow Y$  is called pre  $g\#$ -closed [46] if  $f(V)$  is  $g\#$ -closed set in  $Y$  for each  $g\#$ -closed set  $V$  in  $X$ .

**Theorem 13** If a function  $f: X \rightarrow Y$  is pre  $g\#$ -closed, then for each subset  $B$  of  $Y$  and each  $g\#$ -open set  $U$  of  $X$  containing  $f^{-1}(B)$ , there exists a  $g\#$ -open set  $V$  in  $Y$  containing  $B$  such that  $f^{-1}(V) \subset U$ .

**Proof** Suppose that  $f$  is pre  $g\#$ -closed. Let  $B$  be a subset of  $Y$  and  $U \in G\#O(X)$  containing  $f^{-1}(B)$ . Put  $V = Y \setminus f(X \setminus U)$ , then  $V$  is a  $g\#$ -open set of  $Y$  such that  $B \subset V$  and  $f^{-1}(V) \subset U$ .  
 templateisusedtoformatyourpaperandstylethetext.Allmargins,columnwidths,linespaces,andtextfontsareprescribed;pleasedonotalterthem.Youmaynotpecculiarities.Forexample,theheadmargininthistemplatemeasuresproportionatelymorethaniscustomary.Thismeasurementandothersaredeliberate,usingspecificationsthatanticipateyourpaperasonepartoftheentireproceedings,andnota sanindependentdocument.Pleasedonotrevisenanyofthecurrentdesignations.

#### IV. ( $G\#, s$ )-CONTINUOUS FUNCTIONS

**Definition 14** A function  $f: X \rightarrow Y$  is called ( $g\#, s$ )-continuous if the inverse image of each regular open set of  $Y$  is  $g\#$ -closed in  $X$ .

**Theorem 15** The following are equivalent for a function  $f: X \rightarrow Y$ :

- (1)  $f$  is ( $g\#, s$ )-continuous,
- (2) The inverse image of a regular closed set of  $Y$  is  $g\#$ -open in  $X$ ,
- (3)  $f^{-1}(\text{int}(\text{cl}(V)))$  is  $g\#$ -closed in  $X$  for every open subset  $V$  of  $Y$ ,
- (4)  $f^{-1}(\text{cl}(\text{int}(F)))$  is  $g\#$ -open in  $X$  for every closed subset  $F$  of  $Y$ ,
- (5)  $f^{-1}(\text{cl}(U))$  is  $g\#$ -open in  $X$  for every  $U \in \beta O(Y)$ ,
- (6)  $f^{-1}(\text{cl}(U))$  is  $g\#$ -open in  $X$  for every  $U \in SO(Y)$ ,
- (7)  $f^{-1}(\text{int}(\text{cl}(U)))$  is  $g\#$ -closed in  $X$  for every  $U \in PO(Y)$ .

*Proof*

(1)  $\Leftrightarrow$  (2): Obvious.

(1)  $\Leftrightarrow$  (3): Let  $V$  be an open subset of  $Y$ . Since  $\text{int}(\text{cl}(V))$  is regular open,  $f^{-1}(\text{int}(\text{cl}(V)))$  is  $g\#$ -closed. The converse is similar.

(2)  $\Leftrightarrow$  (4): Similar to (1)  $\Leftrightarrow$  (3)

(2)  $\Leftrightarrow$  (5): Let  $U$  be any  $\beta$ -open set of  $Y$ . By Theorem 2.4 of [2] that  $\text{cl}(U)$  is regular closed. Then by (2)  $f^{-1}(\text{cl}(U))$  is  $g\#$ -open in  $X$ .

(5)  $\Rightarrow$  (6): Obvious from the fact that  $SO(Y) \subset \beta O(Y)$ .

(6)  $\Rightarrow$  (7): Let  $U \in PO(Y)$ . Then  $Y \setminus \text{int}(\text{cl}(U))$  is regular closed and hence it is semi-open. Then we have  $X \setminus f^{-1}(\text{int}(\text{cl}(U))) = f^{-1}(Y \setminus \text{int}(\text{cl}(U))) = f^{-1}(\text{cl}(Y \setminus \text{int}(\text{cl}(U))))$  is  $g\#$ -open in  $X$ . Hence  $f^{-1}(\text{int}(\text{cl}(U)))$  is  $g\#$ -closed in  $X$ .

(7)  $\Rightarrow$  (1): Let  $U$  be any regular open set of  $Y$ . Then  $U \in PO(Y)$  and hence  $f^{-1}(U) = f^{-1}(\text{int}(\text{cl}(U)))$  is  $g\#$ -closed in  $X$ .

**Lemma 16** [35] For a subset  $A$  of a topological space  $(Y, \sigma)$ , the following properties hold:

- (1)  $\alpha \text{cl}(A) = \text{cl}(A)$  for every  $A \in \beta O(Y)$ ,
- (2)  $p \text{cl}(A) = \text{cl}(A)$  for every  $A \in SO(Y)$ ,
- (3)  $s \text{cl}(A) = \text{int}(\text{cl}(A))$  for every  $A \in PO(Y)$ .

**Corollary 17** The following are equivalent for a function  $f: X \rightarrow Y$ :

- (1)  $f$  is ( $g\#, s$ )-continuous,
- (2)  $f^{-1}(\alpha \text{cl}(A))$  is  $g\#$ -open in  $X$  for every  $A \in \beta O(Y)$ ,
- (3)  $f^{-1}(p \text{cl}(A))$  is  $g\#$ -open in  $X$  for every  $A \in SO(Y)$ ,
- (4)  $f^{-1}(s \text{cl}(A))$  is  $g\#$ -closed in  $X$  for every  $A \in PO(Y)$ .

**Proof** It follows from Lemma 16.

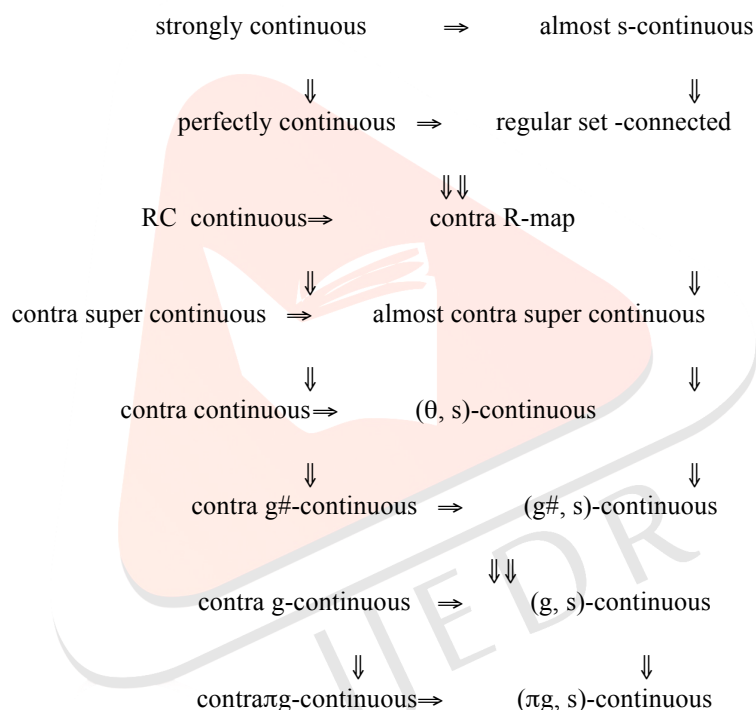
#### V. THE RELATED FUNCTIONS WITH ( $G\#, S$ )-CONTINUOUS FUNCTIONS

**Definition 20** A function  $f: X \rightarrow Y$  is said to be

- (1) perfectly continuous [33] if  $f^{-1}(V)$  is clopen in  $X$  for every open set  $V$  of  $Y$ ,
- (2) regular set -connected [12,16] if  $f^{-1}(V)$  is clopen in  $X$  for every  $V \in RO(Y)$ ,

- (3) almost s-continuous [6,36] if for each  $x \in X$  and each  $V \in SO(Y, f(x))$ , there exists an open set  $U$  in  $X$  containing  $x$  such that  $f(U) \subset scl(V)$ ,
- (4) strongly continuous [24] if the inverse image of every set in  $Y$  is clopen in  $X$ ,
- (5) RC-continuous [11] if  $f^{-1}(V)$  is regular closed in  $X$  for each open set  $V$  of  $Y$ ,
- (6) contra R-map [17] if  $f^{-1}(V)$  is regular closed in  $X$  for each regular open set  $V$  of  $Y$ ,
- (7) contra-super-continuous [22] if for each  $x \in X$  and for each  $F \in C(Y, f(x))$ , there exists a regular open set  $U$  in  $X$  containing  $x$  such that  $f(U) \subset F$ ,
- (8) almost contra-super-continuous [15] if  $f^{-1}(V)$  is  $\delta$ -closed in  $X$  for every regular open set  $V$  of  $Y$ .
- (9) contra continuous [10] if  $f^{-1}(V)$  is closed in  $X$  for every open set  $V$  of  $Y$ ,
- (10) contra g-continuous [5] if  $f^{-1}(V)$  is g-closed in  $X$  for every open set  $V$  of  $Y$ ,
- (11)  $(\theta, s)$ -continuous [23,37] if for each  $x \in X$  and each  $V \in SO(Y, f(x))$ , there exists an open set  $U$  in  $X$  containing  $x$  such that  $f(U) \subset cl(V)$ ,
- (12) contra  $\pi g$ -continuous [19] if  $f^{-1}(V)$  is  $\pi g$ -closed in  $X$  for each open set  $V$  of  $Y$ .
- (13) contra  $g\#$ -continuous [40] if  $f^{-1}(V)$  is  $g\#$ -closed in  $X$  for each open set  $V$  of  $Y$ ,
- (14)  $g\#$ -continuous [46] if  $f^{-1}(V)$  is  $g\#$ -closed in  $X$  for each closed set  $V$  of  $Y$ ,
- (15)  $(g, s)$ -continuous [14] if  $f^{-1}(V)$  is g-closed in  $X$  for each regular open set  $V$  of  $Y$ ,
- (16)  $(\pi g, s)$ -continuous [14] if  $f^{-1}(V)$  is  $\pi g$ -closed in  $X$  for each regular open set  $V$  of  $Y$ .

**Remark 21** The following diagram holds for a function  $f: X \rightarrow Y$ :



None of these implications is reversible as shown in the following examples and in the related papers [14,40].

**Example 22** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\phi, X \setminus \{a\}, \{b, c\}\}$  and  $\sigma = \{\phi, Y, \{b\}, \{a, c\}\}$ . Then the identity function  $f: X \rightarrow Y$  is  $(g, s)$ -continuous but it is not  $(g\#, s)$ -continuous.

**Example 23** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{\phi, X \setminus \{a\}, \{b, c\}\}$  and  $\sigma = \{\phi, Y, \{a, b\}\}$ . Then the identity function  $f: X \rightarrow Y$  is  $(g\#, s)$ -continuous but it is not contra  $g\#$ -continuous.

**Example 24** Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \sigma = \{\phi, X = Y, \{c\}, \{a, d\}, \{a, c, d\}\}$ . Then the function  $f: X \rightarrow Y$  which is defined as  $f(a) = c, f(b) = c, f(c) = b, f(d) = b$  is  $(g\#, s)$ -continuous but it is not  $(\theta, s)$ -continuous.

A topological space  $(X, \tau)$  is said to be extremely disconnected [4] if the closure of every open set of  $X$  is open in  $X$ .

**Definition 25** A function  $f: X \rightarrow Y$  is said to be almost  $g\#$ -continuous if  $f^{-1}(V)$  is  $g\#$ -open in  $X$  for every regular open set  $V$  of  $Y$ .

**Theorem 26** Let  $(Y, \sigma)$  be extremely disconnected. Then, the following are equivalent for a function  $f: (X, \tau) \rightarrow (Y, \sigma)$ :

- (1)  $f$  is  $(g\#, s)$ -continuous,
- (2)  $f$  is almost  $g\#$ -continuous.

*Proof*

(1)  $\Rightarrow$  (2): Let  $x \in X$  and  $U$  be any regular open set of  $Y$  containing  $f(x)$ . Since  $Y$  is extremely disconnected, by lemma 5.6 of [39]  $U$  is clopen and hence  $U$  is regular closed. Then  $f^{-1}(U)$  is  $g^\#$ -open in  $X$ . Thus,  $f$  is almost  $g^\#$ -continuous.

(2)  $\Rightarrow$  (1): Let  $K$  be any regular closed set of  $Y$ . Since  $Y$  is extremely disconnected,  $K$  is regular open and  $f^{-1}(K)$  is  $g^\#$ -open in  $X$ . Thus,  $f$  is  $(g^\#, s)$ -continuous.

**Definition 27** A space  $(X, \tau)$  is called:

- (1)  $T_b[9]$  if every  $g_s$ -closed set is closed.
- (2)  $\pi g-T_{1/2}$  [18] if every  $\pi g$ -closed set is closed.

**Definition 28** A function  $f: X \rightarrow Y$  is called:

- (1) contra  $sg$ -continuous [11] if  $f^{-1}(V)$  is  $sg$ -closed in  $X$  for each open set  $V$  of  $Y$ ,
- (2) contrags-continuous [11] if  $f^{-1}(V)$  is  $g_s$ -closed in  $X$  for each open set  $V$  of  $Y$ .

**Theorem 29** Let  $f: X \rightarrow Y$  be a function from an  $T_b$ -space  $X$  to a topological space  $Y$ . The following are equivalent

- (1)  $f$  is  $(g^\#, s)$ -continuous.
- (2)  $f$  is contra  $g^\#$ -continuous.
- (3)  $f$  is contra  $sg$ -continuous.
- (4)  $f$  is contra  $g_s$ -continuous.
- (5)  $f$  is contra-continuous.

**Proof** Follows by the results in [46].

**Theorem 30** Let  $f: X \rightarrow Y$  be a function from an  $\pi g-T_{1/2}$ -space  $X$  to a topological space  $Y$ . The following are equivalent.

- (1)  $f$  is  $(\theta, s)$ -continuous.
- (2)  $f$  is  $(g^\#, s)$ -continuous.
- (3)  $f$  is  $(g, s)$ -continuous.
- (4)  $f$  is  $(\pi g, s)$ -continuous.
- (5)  $f$  is contra-continuous.

**Definition 31** A space is said to be  $P\Sigma$  [47] or strongly  $s$ -regular [21] if for any open set  $V$  of  $X$  and each  $x \in V$ , there exists  $K \in$

$RC(X, x)$  such that  $x \in K \subset V$ .

**Definition 32** A space  $(X, \tau)$  is called  $g^\#-T_{1/2}$  if every  $g^\#$ -closed set is closed.

**Theorem 33** Let  $f: X \rightarrow Y$  be a function. Then, if  $f$  is  $(g^\#, s)$ -continuous,  $X$  is  $g^\#-T_{1/2}$  and  $Y$  is  $P\Sigma$ , then  $f$  is continuous.

**Proof** Let  $G$  be any open set of  $Y$ . Since  $Y$  is  $P\Sigma$ , there exists a subfamily  $\Phi$  of  $RC(Y)$  such that  $G = \bigcup \{A: A \in \Phi\}$ . Since  $X$  is  $g^\#-T_{1/2}$  and  $f$  is  $(g^\#, s)$ -continuous,  $f^{-1}(G)$  is open in  $X$ . Thus,  $f$  is continuous.

**Theorem 34** Let  $f: X \rightarrow Y$  be a function from a  $\pi g-T_{1/2}$ -space  $(X, \tau)$  to an extremely disconnected space  $(Y, \sigma)$ . Then the following are equivalent.

- (1)  $f$  is  $(\pi g, s)$ -continuous.
- (2)  $f$  is  $(g, s)$ -continuous.
- (3)  $f$  is  $(g^\#, s)$ -continuous.
- (4)  $f$  is  $(\theta, s)$ -continuous.
- (5)  $f$  is almost contra-super-continuous.
- (6)  $f$  is contra  $R$ -map.
- (7)  $f$  is regular set-connected.
- (8)  $f$  is almost  $s$ -continuous.

**Proof** (8)  $\Rightarrow$  (7)  $\Rightarrow$  (6)  $\Rightarrow$  (5)  $\Rightarrow$  (4)  $\Rightarrow$  (3)  $\Rightarrow$  (2)  $\Rightarrow$  (1): Obvious.

(1)  $\Rightarrow$  (8) Let  $V$  be any semi-open and semi-closed set of  $Y$ . Since  $V$  is semi-open,  $cl(V) = cl(int(V))$  and hence  $cl(V)$  is open in  $Y$ . Since  $V$  is semi-closed,  $int(cl(V)) \subset V \subset cl(V)$  and hence  $int(cl(V)) = V = cl(V)$ . Therefore,  $V$  is clopen in  $Y$  and  $V \in RO(Y) \cap RC(Y)$ . Since  $f$  is  $(\pi g, s)$ -continuous,  $f^{-1}(V)$  is  $\pi g$ -open and  $\pi g$ -closed in  $X$ . Since  $X$  is  $\pi g-T_{1/2}$ -space,  $\tau = \pi gO(X)$ . Thus,  $f^{-1}(V)$  is clopen in  $X$  and hence  $f$  is almost  $s$ -continuous [36, Theorem 3.1].

**Definition35** A space is said to be weakly  $P\Sigma$  [34] if for any  $V \in RO(X)$  and each  $x \in V$ , there exists  $F \in RC(X, x)$  such that  $x \in F \subset V$ .

**Theorem 36** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $(g^\#, s)$ -continuous function and Let  $G^\#C(X)$  be closed under arbitrary intersections. If  $Y$  is weakly  $P\Sigma$  and  $X$  is  $g^\#-T^{1/2}$ , then  $f$  is regular set-connected.

**Proof** Let  $V$  be any regular open set of  $Y$ . Since  $Y$  is weakly  $P\Sigma$ , there exists a subfamily  $\Phi$  of  $RC(Y)$  such that  $V = \cup\{A: A \in \Phi\}$ . Since  $f$  is  $(g^\#, s)$ -continuous,  $f^{-1}(V)$  is  $g^\#$ -open in  $X$  for each  $A \in \Phi$  and  $f^{-1}(V)$  is  $g^\#$ -open in  $X$ . Also  $f^{-1}(V)$  is  $g^\#$ -closed in  $X$  since  $f$  is  $(g^\#, s)$ -continuous. Since  $X$  is  $g^\#-T^{1/2}$  space, then  $\tau = G^\#O(X)$ . Hence  $f^{-1}(V)$  is clopen in  $X$  and then  $f$  is regular set-connected.

**Definition 37** A function  $f: X \rightarrow Y$  is said to be  $g^\#$ -irresolute [46] if  $f^{-1}(V)$  is  $g^\#$ -open in  $X$  for every  $V \in G^\#O(Y)$ .

**Theorem 38** Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be functions. Then, the following properties hold:

- (1) If  $f$  is  $g^\#$ -irresolute and  $g$  is  $(g^\#, s)$ -continuous, then  $g \circ f: X \rightarrow Z$  is  $(g^\#, s)$ -continuous.
- (2) If  $f$  is  $(g^\#, s)$ -continuous and  $g$  is contra  $R$ -map, then  $g \circ f: X \rightarrow Z$  is almost  $g^\#$ -continuous.
- (3) If  $f$  is  $g^\#$ -continuous and  $g$  is  $(\theta, s)$ -continuous, then  $g \circ f: X \rightarrow Z$  is  $(g^\#, s)$ -continuous.
- (4) If  $f$  is  $(g^\#, s)$ -continuous and  $g$  is  $RC$  continuous, then  $g \circ f: X \rightarrow Z$  is  $g^\#$ -continuous.
- (5) If  $f$  is almost  $g^\#$ -continuous and  $g$  is contra  $R$ -map, then  $g \circ f: X \rightarrow Z$  is  $(g^\#, s)$ -continuous.

**Theorem 39** Let  $Y$  be regular space and  $f: X \rightarrow Y$  be a function. Suppose that the collection of  $g^\#$ -closed sets of  $X$  is closed under arbitrary intersections. Then if  $f$  is  $(g^\#, s)$ -continuous,  $f$  is  $g^\#$ -continuous.

**Proof** Let  $x$  be an arbitrary point of  $X$  and  $V$  an open set of  $Y$  containing  $f(x)$ . Since  $Y$  is regular, there exists an open set  $G$  in  $Y$  containing  $f(x)$  such that  $cl(G) \subset V$ . Since  $f$  is  $(g^\#, s)$ -continuous, there exists  $U \in G^\#O(X, x)$  such that  $f(U) \subset cl(G)$ . Then  $f(U) \subset cl(G) \subset V$ . Hence,  $f$  is  $g^\#$ -continuous.

## REFERENCES

List and number all bibliographical references in 10-point Times, single-spaced, at the end of your paper. When referenced in the text, enclose the citation number in square brackets, for example: [1]. Where appropriate, include the name(s) of editor(s) of referenced books. The template will number citations consecutively within brackets [1]. This sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in “[3]”—do not use “Ref.[3]” or “reference[3]”. Do not use reference citations as nouns of a sentence (e.g., not: “as the writer explains in [1]”).

Unless there are six authors or more give all authors' names and do not use “etal.”. Paper that have not been published, even if they have been submitted for publication, should be cited as “unpublished” [4]. Paper that have been accepted for publication should be cited as “inpress” [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

- [1] M.E. Abd El-Monsef, S.N. El-Deeb and R.A. Mahmoud,  $\beta$ -open sets and  $\beta$ -continuous mappings, Bull. Fac. Sci. Assiut Univ., 12 (1983), 77-90.
- [2] D. Andrijevic, Semi-preopen sets, Mat. Vesnik, 38 (1986), 24-32.
- [3] S.P. Arya and M.P. Bhamini, Some generalizations of pairwise Urysohn spaces, Indian J. Pure Appl. Math., 18 (1987), 1088-1093.
- [4] N. Bourbaki, General Topology, Part I, Addison Wesley, Reading, Mass 1996.
- [5] M. Caldas, S. Jafari, T. Noiri and M. Simoes, A new generalization of contra continuity via Levine's  $g$ -closed sets, Chaos, Solitons and Fractals, 32 (2007), 1597-1603.
- [6] S.H. Cho, A note on almost  $s$ -continuous functions, Kyungpook Math. J. 4 (2002), 171-175.
- [7] H. Corson and E. Michael, Metrizable of certain countable unions, Illinois J. Math., 8 (1964), 351-360.
- [8] S.G. Crossley and S.K. Hildebrand, Semi-closure, Texas J. Sci., 22 (1971), 99-112.
- [9] R. Devi, H. Maki and K. Balachandran, Semi-generalized closed maps and generalized semi-closed maps. Mem. Fac. Sci. Kochi. Univ. Ser. A. Math. 14 (1993), 41-54.
- [10] J. Dontchev, Contra-continuous functions and strongly  $S$ -closed spaces, Int. J. Math. Math. Sci., 19(2) (1996), 303-310.
- [11] J. Dontchev and T. Noiri, Contra-semicontinuous functions, Math Pannonica, 10 (1999), 159-168.
- [12] J. Dontchev, M. Ganster and I. Reilly, More on almost  $s$ -continuity, Indian J. Math., 41 (1999), 139-146.
- [13] J. Dontchev and T. Noiri, Quasi-normal and  $\pi g$ -closed sets, Acta Math Hungar., 89(3) (2000), 211-219.
- [14] E. Ekici, On  $(g, s)$ -continuous and  $(\pi g, s)$ -continuous functions, Sarajevo Journal of Mathematics, 3 (15) (2007), 99-113.
- [15] E. Ekici, Almost contra-super-continuous functions, Studiis Cercetari Stiintifice, Seria: Matematica, Univ. din Bacau, 14 (2004), 31-42.
- [16] E. Ekici, Properties of regular set-connected functions, Kyungpook Math. J. 44 (2004), 395-403.

- [17] E. Ekici, Another form of contra-continuity, Kochi J.Math.,1 (2006), 21-29.
- [18] E. Ekici and C.W. Baker , On  $\pi g$ -closed sets and continuity, Kochi J. Math. 2 (2007), 35-42.
- [19] E. Ekici. On contra  $\pi g$ -continuous functions, Chaos, Solitons and Fractals, 35 (2008), 71-81.
- [20] E. Ekici, On Contra R-continuity and a weaker form, Indian J. Math. 46 (2-3) (2004), 267-281.
- [21] M. Ganster, On strongly s-regular spaces, Glasnik Mat.,25(45)(1990),195-201.
- [22] S. Jafari and T. Noiri, Contra-super-continuous functions, Ann. Univ. Sci. Buda pest Eotvos Sect. Math.,42 (1999), 27-34.
- [23] J.E. Joseph and M.H. Kwack, On S-closed spaces, Proc. Amer. Math. Soc., 80 (1980), 341-348.
- [24] N. Levine, Strong continuity in topological spaces, Amer. Math. Monthly, 6 (1960), 269.
- [25] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer.MathMonthly ,70 (1963), 36-41.
- [26] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19 (1970), 89-96.
- [27] G.D. Maio, S-closed spaces, S-sets and S-continuous functions, Accad. Sci. Torino, 118 (1984),125-134.
- [28] G.D. Maio and T. Noiri, On s-closed spaces ,Indian J. Pure Appl. Math., 18 (1987), 226-233.
- [29] H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized  $\alpha$ -closed sets and  $\alpha$ -generalized closed sets, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math., 15 (1994), 51-63.
- [30] A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deeb, On precontinuousandweakprecontinuous mappings, Proc.Math.Phys.Soc.Egypt, 53 (1982),. 47-53.
- [31] O. Njastad, On some classes of nearly open sets, Pacific J.Math.,15 (1965), 961-970.
- [32] T. Noiri , On S-closed subspaces, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fiz. Mat Natur.,8 (64) (1978), 157-162.
- [33] T. Noiri, Super-continuity and some strong forms of continuity, Indian J. Pure Appl. Math.,15 (1984), 241-250.
- [34] T. Noiri, A note on S-closed spaces, Bull. Inst. Math. Acad. Sinica,12 (1984), 229-235.
- [35] T. Noiri, On almost continuous functions, Indian J. Pure Appl. Math. 20 (1989),571-576.
- [36] T. Noiri, B. Ahmad and M. Khan, Almost s-continuous functions, KyungpookMath.J., 35 (1995), 311-322.
- [37] T. Noiri and S. Jafari, Properties of  $(\theta, s)$ -continuous functions, Topology Appl.,123 (2002), 167-179.
- [38] T. Noiri, A note on s-regular spaces, Glasnik Mat. 13(33) (1987), 107-110.
- [39] M.C. Pal and P. Bhattacharyya, Faint precontinuous functions, Soochow J. Math.,21(1995),273-289.
- [40] O. Ravi, T. Soupramanien, M.L. Thivagar and R. Nagendran, Contra  $g^\#$ -continuity and separation axioms, Indian J. Math & Math. Sci. 6(1) (2010), 33-40.
- [41] T. Soundararajan, Weakly Hausdorff Spaces and the Cardinality of Topological Spaces ,in: General Topology and its Relation to Modern Analysis and AlgebraIII,Proc. Conf.Kanpur,1968,Academia,Prague 1971,pp.301-306.
- [42] L.A. Steen and J.A. SeebachJr, Counter examples in Topology, Holt, Rinerhartand Winston,New York 1970.
- [43] M.H. Stone, Applications of the theory of Boolean rings to general topology,TAMS, 41 (1937), 375-381.
- [44] T. Thompson, S-closed spaces,P roc.Amer.Math.Soc.,60 (1976), 335-338.
- [45] N.V. Velicko, H-closed topological spaces, Amer. Math. Soc. Transl. 78(1968)103-118.
- [46] M.K.R.S. Veerakumar,  $g^\#$ -closed sets in Topological Spaces, Mem. Fac. Sci Kochi. Univ. (Math.) 24 (2003), 1-13.
- [47] G.J. Wang, On S-closed spaces, Acta Math Sinica, 24 (1981), 55-63.
- [48] V. Zaitsev, On certain classes of topological spaces and their bicompatificationsDokl. Akad. Nauk SSSR,178 (1968), 778-779.