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Flow Of Jeffrey Fluid Through An Artery With Multiple Stenosis

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Abstract - We study the effect of the multiple stenoses on the flow of Jeffrey fluid through an artery with some boundary conditions. Using appropriate boundary conditions the analytical expressions for velocity, flux and volumetric flow rate have been derived. The derived analytical expressions are computed in order to examine the variation of velocity profiles and volumetric flow rate in different regions of flow is discussed and presented graphically.

Keywords - Jeffrey fluid, artery, multiple stenosis, velocity profiles, volume flow rate

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Introduction

The abnormal and unnatural growth in the lumen of an artery is called stenosis. Arteries are narrowed by the development of atherosclerotic plaques that protrude into the lumen, resulting arterial stenosis. This led to extensive investigations of the characteristics of blood flow through a stenosed artery in recent years. Yound (1968) was one amongst the first who attempted to theoretically analyse the interaction of mild stenosis with fluid dynamics of blood flow. Similar problems were subsequently investigated by many other researches like Shukla et al (1980), Chaturani and Samy (1985) to understand the flow patterns in stenosed arteries. In their studies, blood has been characterised as Newtonian fluid and little attention has been given to its suspension nature. An unsteady analysis of non-Newtonian blood flow through tapered arteries with a stenosis was studied by Mandal
(2005).

Similar studies were done by Sankar and Hemalatha (2006) in Pulsatie flow o fHerschel-Bulkley fluid through stenosed artiries. Here the effects of pulsatility, stenosis and non-Newtonian behaviour of blood are considered. A perturbation method is used to analyse the flow assuming the thickness of plug core region to be non-uniform changing with axial distance. Also the variation of wall shear stress distribution and resistance to flow with axial distance for different values of time and for different values of yield stress are analysed. Also Moayeri and Zendehbudi (2003) have discussed the effects of elastic property of the wall on flow characteristics through arterial stenoses. Here hemodynamic characteristics of blood flow through arterial stenosis are investigated. Blood is assumed as a Newtonian fluid and the pulsatile nature of flow is modelled by using measured values of the flow rate and pressure for the canine femoral arteries. The result indicate that deformability of the wall causes an increase in the time average of pressure drop, but a decrease in the maximum wall shear stress.

Two dimensional blood flow through tapered arteries under stenotic conditions was analysed by Chakravarty and Mandal (2000). Long et al. (2001) investigated pulsatile flow through arterial stenosis numerically. Numerical simulation of pulsatile blood flow in straight tube stenosis model was performed to investigate the post stenotic flow phenomena. Flow features such as velocity profiles, flow separation zone and wall shear stress distribution in the post stenotic region are described. Results shows that the formation and development of flow separation zone in the post stenotic region are very complex. Flow n a catheterised curved artery with stenos was studied by Dash et al. (1999). Here the investigation is done through mathematical analysis. Blood is modelled as an incompressible Newtonian fluid and the flow is assumed to be steady and laminar. The effect of Catheterization on various physiologically important flow characteristics is studied for different values of catheter size and Reynolds number of the flow.

Tu and Deville (1966) was studied Pulsatile flow of non-Newtonian fluids through artierial stenosis. Here the blood flow through stenosis is solved using the incompressible generalised Newtonian model. The Hershel-Bulkley. Bingham and Power law fluids are incorporated. Kanpur (1992) has reported yield stress and they are more suitable for the studies of the blood flow through narrow arteries. A survey of the literature on arteriosclerotic development indicates that the studies conducted are mainly concerned with single symmetric and non-symmetric stenosis. The stenosis may develop in series (multiple stenoses) or may be of the irregular shape or overlapping. Effects of an overlapping stenosis on artierial flow problem of blood, assuming the pressure is varying along the axis of the tube was studied by Chakravarty and Mandal (1994). A remarkable new shape of the stenosis in the region of the formation of the arterial narrowing caused by a thermo is constructd mathematically. The artery is simulated as an elastic cylindrical tube containing a viscoelastic fluid representing blood through quantitative analysis is performed for the flow velocity, flux, the resistive impedances and the wall shear stress with their variation with time.

Sinha and Singh (1984) and Srivastava (1985) considered a couple stress model for blood flow through stenosed tube to account for the size effects of the suspended particles. For a fixed stenosis size the resistance to flow and wall shear stress increase as the couple stress parameter decreases from unity. A comparision of the results with those of the Newtonian case shows that the magnigtude of resistance to flow and wall shear under a given set of condition is greater in the case of couple stress fluid model. Also Parvathamma and Devanathan (1985) used a micro polar fluid model to account for particles spin while Prahlad and Shultz (1988) discussed the blood flow through a stenosed artery by considereing polar fluid model for blood.

Shukla and Parihar (1980) has analysed the effect of stenosis on non-Newtonian flow of the blood in an artery, it has been shown that the resistance of flow and the wall shear increase with the size of stenosis but these increase are comparatively small due to non-Newtonian behaviour of the blood indicating the usefulness of its rheological character in the functioning of the diseased arterial circulation.

Nakamura and Sawada (1988) analysed numerically the flow of a non-Newtonian fluid through an axi-symmetric stenosis. The flow pattern, the separation and reattachement points and the distribution of pressure and shear stress at the wall are obtained. The axial force acting on the stenosis is evaluated and this force becomes one of the causes of post stenotic dilation. Also it shows that the non-Newtonian property of blood weakens the distortion of flow pattern, pressure and shear stress at the wall associated with the stenosis.

A study related to a non-Newtonian fluid model for blood flow through artiries under stenotic condition was given by Misra et al. (1993). This is an analytical study on the behaviour of blood flow through an arterial segment having a mild stenosis. The artery has bee treated as a thin walled initially stresses orthotropic non-linear viscous elastic cylindrical tube filled with a non-Newtonian fluid representing blood. It has been shown that the resulting analytical expressions that the resistance to flow and the wall shear increase as the size of the stenosis increases.

Lioyd et al. (1977) analysed the pulsatile flow of the blood through diseased coronary arteries of man. The rheological effects of multiple, non-obstructive plaques in main coronary arteries of man were examined by numerically solving the fluid dynamics equations of motion for pulsatile viscous flow of blood through an arterial section using the actual variation of flow rate during the cardiac cycle. Yet another study on the effect of non-Newtonian property of blood on flow through a stenosed tube was given by Takuji et al. (1998). Here periodic blood flow through a stenosed tube analysed numerically. Fluid dynamics of arterial blood flow plays an important role in arterial diseases. The bi-viscosity model is used as a constitutive equation for blood and flow is assumed as periodic, incompressible and axi-symmetric. The result show that the non-Newtonian property reduces the strength of the vortex down stenosis and has considerable influence in the flow even at high stokes. A mathematical model of flow through an irregular arterial stenosis is developed by Peter and David Kilpatrick (1991). This model over estimates the pressure drops across the stenosis as the wall shear stress and separation. Perktold et al. (1991) analysed numerically the flow and stress patterns in human carotid artery bifurcation models which differ in the bifurcation angle. Under physiologically relevant flow conditions the study concentrates on flow and stress characteristics in the carotid sinus. Halder (1985) studied the effects of the shape of stenosis on the resistance to blood flow through an artery with mild local narrowing. The resistance to flow decreases as blood flow through an artery with mild local narrowing. The resistance to flow decreases as the shape of the stenosis changes and maximum resistance is attained in the case symmetric stenosis. Blood flow through an axi-symmetric stenosis studied by Pontrelli (2001). A shear thinning fluid modelling the deformation – dependent viscosity of the blood is proposed. The motion equation is written in vorticity – steam function formulation and is solved numerically by a finite difference scheme.

Siddiqui et al. (2009) developed a mathematical modelling of pulsatile flow of cassons's fluid in arterial stenosis. An important result is that the mean and steady flow rates decreases as the yield stress increases. Another important result of pulsatility is the mean resistance to flow is greater than the its steady flow is equal to steady wall shear stress.

Mandal et al. (2007) has developed a mathematical model by treating blood as a non-Newtonian fluid characterised by the generalised power-law model incorporating both the shear-thinning and shear-thickening characters of the streaming blood. The arterial wall has been treated as an elastic cylindrical tube having a stenosis its lumen. The unsteady flow mechanism in the stenosed artery subject to a pulsatile pressure gradient arising from the normal functioning of heart has been accounted for.

Zheng Lou and Wen – Jei Yang(1993) analysed the effect of the non-Newtonian behaviour of blood on a pulsatile flow at the aortic bifurcation. The blood rheology was described by a weak form of Casson equation. It was disclosed that the non-Newtonian property of blood did not drastically change the flow patterns, but caused an increase in shear stresses and a slightly higher resistance to both flow separation.

The assumption of Newtonian behaviour of blood is acceptable for high shear rate flow through larger arteries. Tu and Deville (1996), has analysed thepulsatile flow of non-Newtonian fluids through arterial stenoses. But blood, being a suspension of cells in plasma, exhibits non-Newtonian behaviour at low shear rate in small diameter arteries (Chien 1981). In diseased state, the actual flow is distinctly pulsatile was studied by Srivastava and Saxena (1994). Chaturani and Samy (1986), has investigated the pulsatile flow of Casson's fluid through stenosed arteties with application to blood flow. A perturbation method is uses to analyse the flow. For the low values of the yield stress, the mean flow rate behaviour is of opposite nature. It is reported that the rheological properties of blood and its flow behaviour through tubes of varying cross section play an important role in understanding the diagnosis and treatment of many cardiovascular diseases. The pulsatile flow of blood through stenosed arteries is analysed by assuming the blood as a two fluid model with the suspension of all the erythrocytes in core region as a non-Newtonian fluid and the plasma in the peripheral layer as a Newtonian fluid. The non-Newtonian fluid in the core region is assumed as (1) Hershel-Bulkley fluid and (2) Casson fluid. It is found that the plug flow and the resistance to flow are very low for Casson fluid. It has been established by Menil et al. (1965), that Casson model would be more useful to analyse the blood flow through stenosed artieries or tubes of diameter 130-1300 m. Jeffrey fluid also exhibits the characteristic of non-Newtonian fluids like Casson fluid and Hershel-Bulkley fluids.

In the present article we study the effect of the multiple stenoses on the flow of a Jeffrey fluid in an artery. The analytical results for velocity and flux are derived. The derived analytical expressions are computed in order to examine the variation of velocity profiles and volumetric flow rate in different regions of flow.

Mathematical formulation:

Consider the steady flow of a Jeffrey fluid through a tube with non-uniform cross section and with two stenoses. We consider the cylindrical polar co-ordinate system (r, z so that z coincides with the centre line of the channel. The stenoses are mild and axially symmetric.

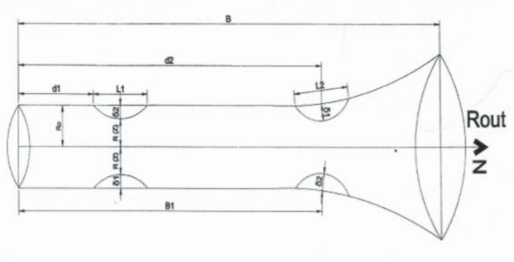

Fig 1:- Physical model

The radius of the tube is taken as h(z) = R(z) and

$R(z) = R_0$ where $0 \le z \le d_1$

$= R_0 - \frac{\delta_1}{2}\left\{1 + \cos\frac{2\pi}{L_1}\left(z - d_1 - \frac{L_1}{2}\right)\right\}$ $\quad d_1 \le z \le d_1 + L_1$

$= R_0$ $\quad d_1 + L_1 \le z \le B_1 - \frac{L_1}{2}$

$= R_0 - \frac{\delta_2}{2}\left\{1 + \cos\frac{2\pi}{L_2}\left(z - B_1 - \frac{L_2}{2}\right)\right\}$ $\quad B_1 - L_2 \le z \le B$

$= R^*(z) - \frac{\delta_2}{2}\left\{1 + \cos\frac{2\pi}{L_2}\left(z - B_1 - \frac{L_2}{2}\right)\right\}$ $\quad B_1 \le z \le B_1 + \frac{L_2}{2}$

$= R^*(z)$ $\quad B_1 + \frac{L_2}{2} \le z \le B$ ----------(1)

Here L_i and δ_i (i=1,2) are the lengths and maximum thickness of two stenosis respectively and the restrictions for the mild stenosis are satisfied.

$\delta_i << \min(R_0, R_{out})$

$\delta_i << L_i (i = 1,2)$

Where $R_{out} = R(z)$ at $z = B$

The basis equation governing the flow is

$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{1+\lambda_1}\frac{\partial w}{\partial r}\right) = -\frac{1}{\mu}\frac{\partial p}{\partial z}$ ------------(2)

Where λ_1 is Jeffrey parameter, p is pressure, μ is viscosity of the fluid, R_0 is the radius of the tube.

The boundary conditions are

$\frac{\partial w}{\partial r} = 0$ when r = 0 -----------(3)

W = 0 when r = R(z) --------(4)

Introducing the following non-dimensional variables

$\bar{r} = \frac{r}{R_0}$ $\quad \bar{p} = \frac{pR_0^2}{\mu UB}$ $\quad \bar{d} = \frac{d_1}{B}$

$\overline{L_2} = \frac{L_2}{B}$ $\quad \bar{w} = \frac{w}{\mu}$ $\quad \bar{z} = \frac{z}{B}$

$\overline{L_1} = \frac{L_1}{B}$ $\quad \overline{B_1} = \frac{B_1}{B}$ $\quad \bar{R}(z) = \frac{R(z)}{R_0}$

$\bar{Q} = \frac{Q}{\pi U R_0^2}$ $\quad \bar{\delta_i} = \frac{R(z)}{R_0}$ ------------ (5)

Non-dimensionalising the governing equations after dropping bars

$\frac{\partial}{\partial r}\left(\frac{r}{1+\lambda_1}\frac{\partial w}{\partial r}\right) = r\frac{\partial p}{\partial z}$ ------------(6)

The non- dimensional boundary conditions are

$\frac{\partial w}{\partial r} = 0$ when r = 0 -----------(7)

W = 0 when r = R(z) ----------(8)

Solution of the problem

(i) Velocity Distribution

Integrating eqn (1) by applying boundary conditions (7) and (8) the axial velocity can be obtained as

$w = \frac{1}{4(1+\lambda_1)}\frac{\partial p}{\partial z}(R^2 - r^2)$ --(9)

The volumetric flow rate is obtained as

$Q = \frac{1}{16(1+\lambda_1)}\frac{\partial p}{\partial z}R^4(z)$ ---(10)

Pressure Difference

The pressure difference Δp along the total length of the tube as follows

$\Delta p = \int_0^1 \frac{Q\,16(1+\lambda_1)}{R^4(z)}$ ------------(11)

Results and discussions

From equation(9) we have calculated the axial velocity as a function of r for different values of Jeffrey parameter λ_1 in the stenotic regions $d_1 \le z \le d_1 + L_1$ and $B_1 - \frac{L_2}{2} \le z \le B_1$ and is shown in figures (2) and (3). It is observed that the velocity decreases with the increase in the Jeffery parameter $\lambda_1$in both the stenotic regions.

The volumetric flux is calculated from equation (10) for different Jeffery parameter λ_1 in the stenotic region $d_1 \le z \le d_1 + L_1$ and is shown in figure (4). The curve resemble an inverted parabola. It is noticed that minimum flux rate is attained at z = 0.3, that is the mid point of the stenosis.

Corresponding to the stenotic region $B_1 - \frac{L_2}{2} \le z \le B_1$ the volumetric flux is calculated for different Jeffery parameter and is shown in figure (5). It is observed that the flux rate decreases with increase in Jeffery parameter λ_1 in the stenotic region.

The volumetric flux is calculagted for different values of **k** and is shown in figure (6). In the third stenotic region for numerical computation we assume

$$\frac{R^*(z)}{R_0} = e^{[k(z - B_1)^2} = e^{[\bar{k}(\bar{z} - \overline{B_1})^2} \text{ . where } \bar{k} = kB^2$$

And $\bar{d}_1 = 0.2, \overline{L_1} = 0.2, \overline{B_1} = 0.2, \overline{L_2} = 0.8$. Here we observed that flux increases in the value of **k.**

The varaiation of Q the flow rate for different values of Jeffery parameter λ_1 in the third stenotic region $B_1 \le z \le B_1 + \frac{L_2}{2}$ is shown in figure (7). Here it is observed that flux decreases as Jeffery parameter λ_1 increases.

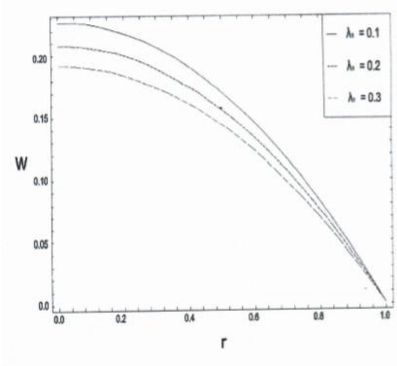

Fig 2: Velocity profiles for different Jeffrey Parameter λ_1 in the first stenotic region $d_1 \le z \le d_1 + L_1$

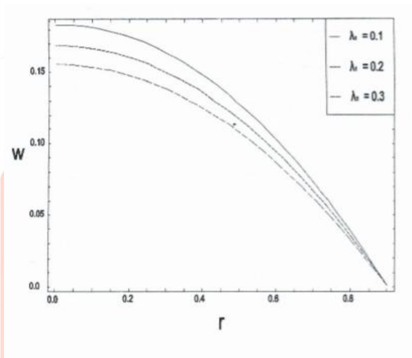

Fig 3: Velocity profiles for different Jeffrey Parameter λ_1 in the second stenotic region $B_1 - \frac{L_2}{2} \le z \le B_1$

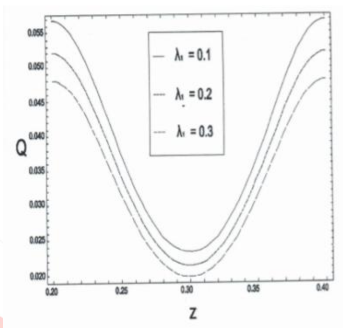

Fig 4: Volumetric Flux for different Jeffrey parameter λ_1 in the first stenotic region $d_1 \le z \le d_1 + L_1$

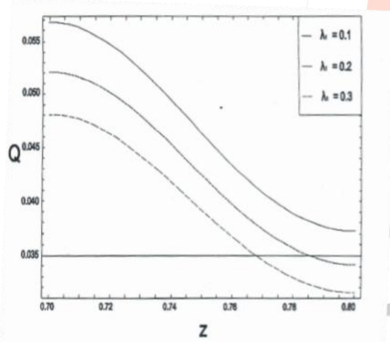

Fig 5: Volumetric Flux for different Jeffrey parameter λ_1 in the second stenotic region $B_1 - \frac{L_2}{2} \le z \le B_1$

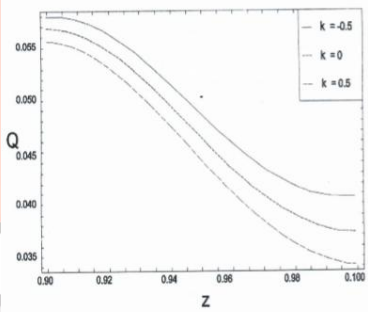

Fig 6: Volumetric Flux for different k in the third stenotic region $B_1 \le z \le B_1 + \frac{L_2}{2}$

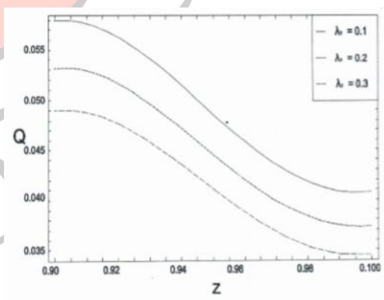

Fig 7: The Volumetric Flux for different Jeffrey parameter λ_1 in the third stenotic region $B_1 \le z \le B_1 + \frac{L_2}{2}$

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