

# Finite element analysis of Funicular shell footing on Winkler foundation

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**Abstract** - Shells, because of their curved topology, possess larger stiffness and strength compared to corresponding plane surface structural elements. This kind of form enables shells to put a minimum of material to maximum structural advantages. Shell footings have been found to be economical foundations in areas having high material to labor cost ratio. Shell footing has greater load carrying capacity compared with flat shallow foundations. Moreover, shells are essentially thin structures, thus structurally more efficient than flat structures. This is an advantage in situations involving heavy super structural loads to be transmitted to weaker soils. In the present study multiple Funicular shaped Shell Footings are investigated. For modelling of Shell structure and its analysis, a computer software, Staad Pro v8i is used. Funicular shell footings are generated in Staad Pro using 4 noded rectangular elements and soil is modelled as elastic springs in compression only with modulus of sub grade reaction taken as 12000 KN/m<sup>2</sup>/m. Complete parametric study is carried out for different cases of footings with different Shell rises, thicknesses and edge lengths.

**keywords** - Funicular Shell Footings, Finite element analysis, Elastic Springs, Sub grade Modulus

## I. INTRODUCTION

### GENERAL

In last few years, curved concrete shell structures in general practices seem to have gained popularity, probably because of the vast developments in computer modelling technology. Nowadays literally all kind of shapes can be easily drawn by the help of Computer Aided Design (CAD) software and can be analyzed by advanced Finite Element modelling (FEM) methods.

### BEHAVIOR OF CONCRETE SHELLS

Shells, because of their curved topology, possess larger stiffness and strength than compared to corresponding plane surface structural elements. This kind of form enables shells to put a minimum of material to maximum structural advantages. While a plane element similar to a roof slab undergoes bending when subjected to vertical loads including self-weight, a shell which is non-planar or a spatial system sustains the applied loads primarily by direct in-plane or membrane forces (compression or tension). A shell may balance an applied transverse loading at the expense of the membrane stresses mainly, with bending actions minimized. As mentioned above, shell thickness demonstrates the high efficiency of shells. It is associated with the shell's low weight and simultaneously its high strength.

### SHELLS IN ENGINEERING

Thin shells as structural elements occupy a leadership position in engineering and, in particular, in civil, mechanical, architectural, aeronautical, and marine engineering. Examples of shell structures in civil and architectural engineering are large-span roofs, liquid-retaining structures and water tanks, containment shells of nuclear power plants, and concrete arch domes. In mechanical engineering, shell forms are used in piping systems, turbine disks, and pressure vessels technology.

The wide application of shell structures in engineering is conditioned by their following advantages:

1. Efficiency of load-carrying behavior.
2. High degree of reserved strength and structural integrity.
3. High strength: weight ratio. This criterion is commonly used to estimate structural component efficiency: the larger this ratio, the more optimal is a structure. According to this criterion, shell structures are much superior to other structural systems having the same span and overall dimensions.
4. Very high stiffness.
5. Containment of space.

### FOUNDATION

Most of the structures built by us are made of reinforced concrete. Here, the part of the structure above ground level is called as the super structure, where the part of the structure below the ground level is called as the substructure. Footings are located below the ground level and are also referred as foundations. Footings transfer the vertical loads, Horizontal loads, Moments, and other forces to the soil.

The important purpose of foundation are as follows;

1. To transfer forces from superstructure to firm soil below.
2. To distribute stresses evenly on foundation soil such that foundation soil neither fails nor experiences excessive settlement.

3. To develop an anchor for stability against overturning.
  4. To provide an even surface for smooth construction of superstructure.
- Due to the loads and soil pressure, footings develop Bending moments and Shear forces.  
Calculations are made as per the guidelines suggested in IS 456 2000 to resist the internal forces.

**SHELLS AS FOUNDATIONS**

An economic alternative to traditionally plain shallow foundations especially where heavy super structural loads are to be transmitted to weaker soil is opportune incentive to use shell foundations. Shell footings as foundations rely heavily on their geometrical shape and streamlined continuity to induce strength and perform efficiently in soil.

**II. OBJECTIVES OF THE STUDY**

Having conducted an exhaustive literature review and in lieu of the premise for improved soil–structure performance stemming from a comprehensive investigation into inverted shell foundation footings, the objectives of the present research are:

- a) To study the geotechnical behavior of shells performance in terms of bearing capacity and settlement and determine existing boundary of knowledge on the topic
- b) To develop foundation model configurations which will produce a more uniform contact pressure distribution and conceivably optimize structural shell design (i.e. achieve higher bearing capacities and produce less settlement than existing foundation designs)
- c) To plot load verses settlement graphs and evaluate the performance of Funicular shell footing.
- d) To perform complete parametric study on funicular shell footing with different shell rises, shell warps and thickness.
- e) To develop a theory for Funicular shell footing foundation performance based on the soil behavioral response by predicting the general rupture surface utilizing bearing capacity coefficients for this case
- f) To promote shell footings as an economic alternative to conventional foundations
- g) To plot graphs for bending moment and shear forces along the diagonal.

**III. NUMERICAL SHELL MODELING**

Numerical shell modelling basically provides an advantage over conventional experimental models. Modelling of the whole system is more preferable in terms of economy and time. Moreover in costly physical tests the accuracy of the procedure cannot be certain and it may not be correct due human errors, instrumental errors, material error etc. and give false results. In the present study the whole system is modelled using STAAD PRO V8i.

**Shell Model Generation and Equation**

All Shell models are generated in Staad Pro using 4 noded rectangular elements.  
Hyper paraboloid shells on rectangular plan are generated using equation as below:

$$z = \frac{z_{Max}}{a \cdot b} (x)(y)$$

And Funicular shells on rectangular plan are generated using equation as below:

$$z = \frac{z_{Max}}{a^2 \cdot b^2} (a^2 - x^2)(b^2 - y^2)$$

Shell co-ordinates are first generated using Microsoft Excel and then these co-ordinates are given in Staad pro for all nodes. Rectangular Plate elements are used for the whole structure with specified properties.

For the purpose of validation a hypar paraboloid shell footing used by Dr. Adel A. Al-Azzawi, Dr. Riyadh J. Aziz, Ali A. Al-Ani on Finite element analysis of hypar paraboloid shell footing is generated.

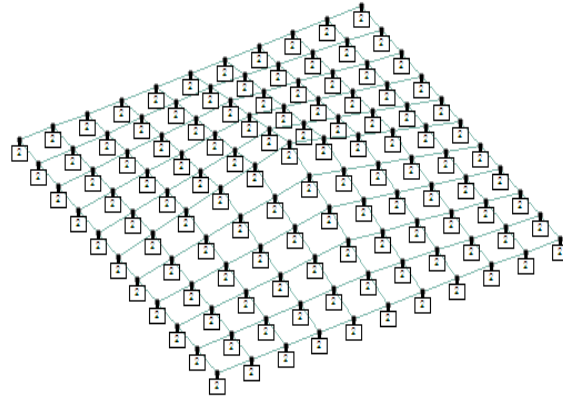
Following cases of shell footings are analyzed:-

S.No.	Type	Plan (m)	Rise (m)	Thickness (mm)
1	Hyparabolic paraboloid	5 x 5	0.5	100
2	Funicular	5 x 5	0.25	100
3	Funicular	5 x 5	0.5	100
4	Funicular	5 x 5	0.75	100
5	Funicular	5 x 5	1	100
6	Funicular	5 x 5	1.5	100
7	Funicular	5 x 5	2	100
8	Funicular	5 x 5	0.5	50
9	Funicular	5 x 5	0.5	75
10	Funicular	5 x 5	0.5	125
11	Funicular	5 x 5	0.5	150
12	Funicular	3 x 3	0.25	100
13	Funicular	4 x 4	0.25	100

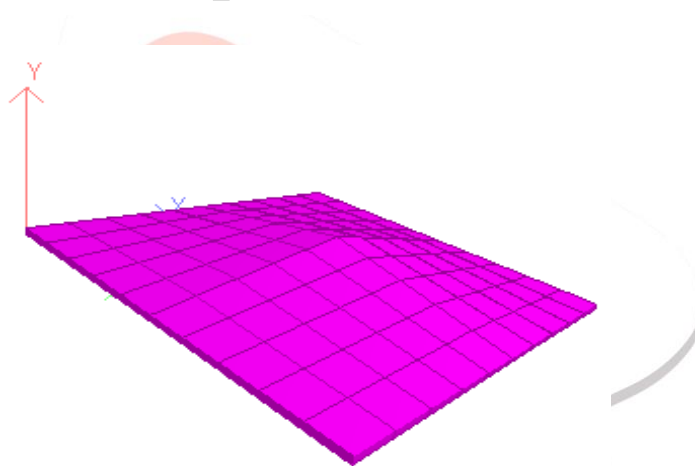
14	Funicular	6 x 6	0.25	100
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Generating mesh, 3-D model and stress contour of hyperbolic paraboloid footing, all funicular footings and Flat footing is shown as below:

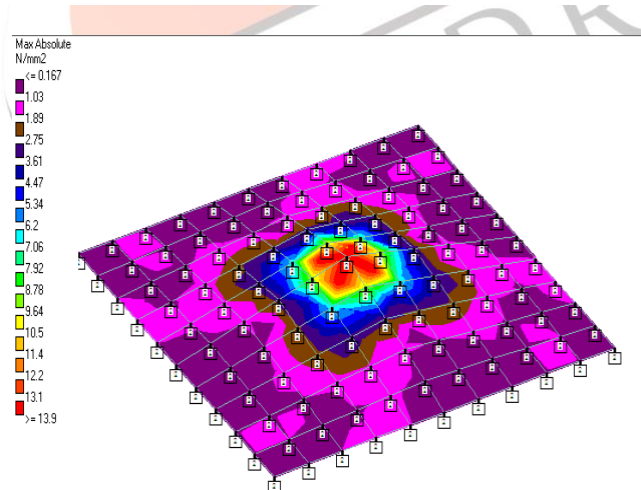
Hypar paraboloid 5x5 rise 0.5 and thickness 100mm  
 Finite element mesh structure



3-D Model



Stress contour



**IV. SOIL STRUCTURE MODELLING**

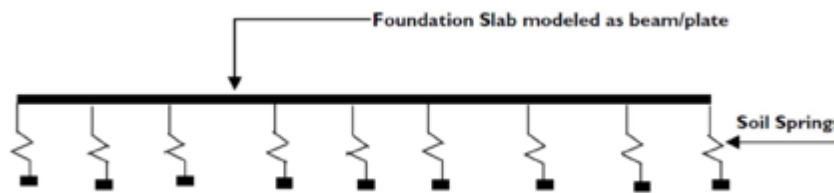
Soil is modelled using the concept of Winkler’s springs.

Winkler’s springs basically represents the soil strata as a system of identical but mutually independent, closely spaced, discrete, linearly elastic springs.

According to this idealization, deformation of foundation due to applied load is confined to loaded regions only.

The pressure–deflection relation at any point is given by  $p = k * w$ , where  $k$  = modulus of sub grade reaction.

Figure shows the physical representation of the Winkler foundation.



In the present study the value of Sub grade reaction is provided as 12000 Kn/m<sup>2</sup>/m for all cases which represent medium dense sand case.

Spring supports are provided to all nodes acting as an elastic mat with given sub grade and modeled for compression only.

**V. ANALYSIS OF SHELL FOOTING**

Static linear analysis is carried out for determination of stress conditions.

**TEST FOOTING**

Shell Properties

Property	
Material	Concrete
Compressive strength(fcu)	27.58 N/mm <sup>2</sup>
Elastic Modulus	21718 N/mm <sup>2</sup>
Density	2402.615 Kg/m <sup>3</sup>
Poission’s ratio	0.170
Alpha/degree c	10E-6
Damping	0.05
Isotropic concrete	

Funicular 5x5 rise 0.5 m

Hypar paraboloid 5x5 rise 0.5 m

Node no.	X	Y	Z	Node no.	X	Y	Z
1	0	0.5	0	1	0	0	0
2	2.5	0	0	2	2.5	0	0
3	0	0	2.5	3	0	0	2.5
4	2.5	0	2.5	4	2.5	0.5	2.5
5	0.5	0.48	0	5	0.5	0	0
6	0.5	0.4608	0.5	6	0.5	0.02	0.5
7	0	0.48	0.5	7	0	0	0.5
8	1	0.42	0	8	1	0	0
9	1	0.4032	0.5	9	1	0.04	0.5
10	1.5	0.32	0	10	1.5	0	0
11	1.5	0.3072	0.5	11	1.5	0.06	0.5
12	2	0.18	0	12	2	0	0
13	2	0.1728	0.5	13	2	0.08	0.5
14	2.5	0	0.5	14	2.5	0.1	0.5
15	0.5	0.4032	1	15	0.5	0.04	1
16	0	0.42	1	16	0	0	1
17	1	0.3528	1	17	1	0.08	1
18	1.5	0.2688	1	18	1.5	0.12	1
19	2	0.1512	1	19	2	0.16	1
20	2.5	0	1	20	2.5	0.2	1
21	0.5	0.3072	1.5	21	0.5	0.06	1.5
22	0	0.32	1.5	22	0	0	1.5
23	1	0.2688	1.5	23	1	0.12	1.5
24	1.5	0.2048	1.5	24	1.5	0.18	1.5
25	2	0.1152	1.5	25	2	0.24	1.5
26	2.5	0	1.5	26	2.5	0.3	1.5
27	0.5	0.1728	2	27	0.5	0.08	2
28	0	0.18	2	28	0	0	2

29	1	0.1512	2	29	1	0.16	2
30	1.5	0.1152	2	30	1.5	0.24	2
31	2	0.0648	2	31	2	0.32	2
32	2.5	0	2	32	2.5	0.4	2
33	0.5	0	2.5	33	0.5	0.1	2.5
34	1	0	2.5	34	1	0.2	2.5
35	1.5	0	2.5	35	1.5	0.3	2.5
36	2	0	2.5	36	2	0.4	2.5
37	0	0	-2.5	37	0	0	5
38	2.5	0	-2.5	38	2.5	0	5
39	0	0.48	-0.5	39	0	0	4.5
40	0.5	0.4608	-0.5	40	0.5	0.02	4.5
41	0	0.42	-1	41	0.5	0	5
42	0.5	0.4032	-1	42	0	0	4
43	0	0.32	-1.5	43	0.5	0.04	4
44	0.5	0.3072	-1.5	44	0	0	3.5
45	0	0.18	-2	45	0.5	0.06	3.5
46	0.5	0.1728	-2	46	0	0	3
47	0.5	0	-2.5	47	0.5	0.08	3
48	1	0.4032	-0.5	48	1	0.04	4.5
49	1	0.3528	-1	49	1	0	5
50	1	0.2688	-1.5	50	1	0.08	4
51	1	0.1512	-2	51	1	0.12	3.5
52	1	0	-2.5	52	1	0.16	3
53	1.5	0.3072	-0.5	53	1.5	0.06	4.5
54	1.5	0.2688	-1	54	1.5	0	5
55	1.5	0.2048	-1.5	55	1.5	0.12	4
56	1.5	0.1152	-2	56	1.5	0.18	3.5
57	1.5	0	-2.5	57	1.5	0.24	3
58	2	0.1728	-0.5	58	2	0.08	4.5
59	2	0.1512	-1	59	2	0	5
60	2	0.1152	-1.5	60	2	0.16	4
61	2	0.0648	-2	61	2	0.24	3.5
62	2	0	-2.5	62	2	0.32	3
63	2.5	0	-0.5	63	2.5	0.1	4.5
64	2.5	0	-1	64	2.5	0.2	4
65	2.5	0	-1.5	65	2.5	0.3	3.5
66	2.5	0	-2	66	2.5	0.4	3
67	-2.5	0	0	67	5	0	5
68	-2.5	0	-2.5	68	5	0	2.5
69	-0.5	0.48	0	69	4.5	0	5
70	-0.5	0.4608	-0.5	70	4.5	0.02	4.5
71	-1	0.42	0	71	5	0	4.5
72	-1	0.4032	-0.5	72	4	0	5
73	-1.5	0.32	0	73	4	0.04	4.5
74	-1.5	0.3072	-0.5	74	3.5	0	5
75	-2	0.18	0	75	3.5	0.06	4.5
76	-2	0.1728	-0.5	76	3	0	5
77	-2.5	0	-0.5	77	3	0.08	4.5
78	-0.5	0.4032	-1	78	4.5	0.04	4
79	-1	0.3528	-1	79	5	0	4
80	-1.5	0.2688	-1	80	4	0.08	4
81	-2	0.1512	-1	81	3.5	0.12	4
82	-2.5	0	-1	82	3	0.16	4
83	-0.5	0.3072	-1.5	83	4.5	0.06	3.5
84	-1	0.2688	-1.5	84	5	0	3.5
85	-1.5	0.2048	-1.5	85	4	0.12	3.5
86	-2	0.1152	-1.5	86	3.5	0.18	3.5
87	-2.5	0	-1.5	87	3	0.24	3.5
88	-0.5	0.1728	-2	88	4.5	0.08	3
89	-1	0.1512	-2	89	5	0	3

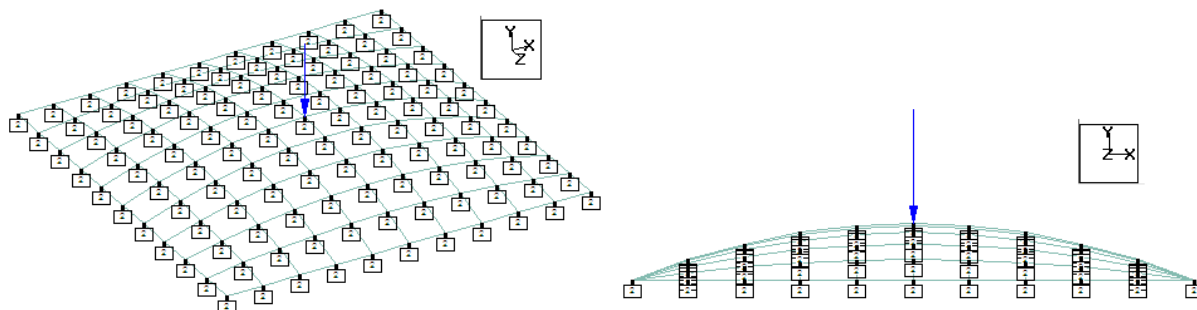
90	-1.5	0.1152	-2	90	4	0.16	3
91	-2	0.0648	-2	91	3.5	0.24	3
92	-2.5	0	-2	92	3	0.32	3
93	-0.5	0	-2.5	93	4.5	0.1	2.5
94	-1	0	-2.5	94	4	0.2	2.5
95	-1.5	0	-2.5	95	3.5	0.3	2.5
96	-2	0	-2.5	96	3	0.4	2.5
97	-2.5	0	2.5	97	5	0	0
98	-0.5	0.4608	0.5	98	5	0	0.5
99	-0.5	0.4032	1	99	4.5	0.02	0.5
100	-0.5	0.3072	1.5	100	4.5	0	0
101	-0.5	0.1728	2	101	5	0	1
102	-0.5	0	2.5	102	4.5	0.04	1
103	-1	0.4032	0.5	103	5	0	1.5
104	-1	0.3528	1	104	4.5	0.06	1.5
105	-1	0.2688	1.5	105	5	0	2
106	-1	0.1512	2	106	4.5	0.08	2
107	-1	0	2.5	107	4	0.04	0.5
108	-1.5	0.3072	0.5	108	4	0	0
109	-1.5	0.2688	1	109	4	0.08	1
110	-1.5	0.2048	1.5	110	4	0.12	1.5
111	-1.5	0.1152	2	111	4	0.16	2
112	-1.5	0	2.5	112	3.5	0.06	0.5
113	-2	0.1728	0.5	113	3.5	0	0
114	-2	0.1512	1	114	3.5	0.12	1
115	-2	0.1152	1.5	115	3.5	0.18	1.5
116	-2	0.0648	2	116	3.5	0.24	2
117	-2	0	2.5	117	3	0.08	0.5
118	-2.5	0	0.5	118	3	0	0
119	-2.5	0	1	119	3	0.16	1
120	-2.5	0	1.5	120	3	0.24	1.5
121	-2.5	0	2	121	3	0.32	2

**Support properties**

Type	Winkler foundation
Foundation	Elastic Mat
Modullus of sub grade reaction	12000 KN/m2/m
Direction	Y only
Spring	Compression only

**Load assignment**

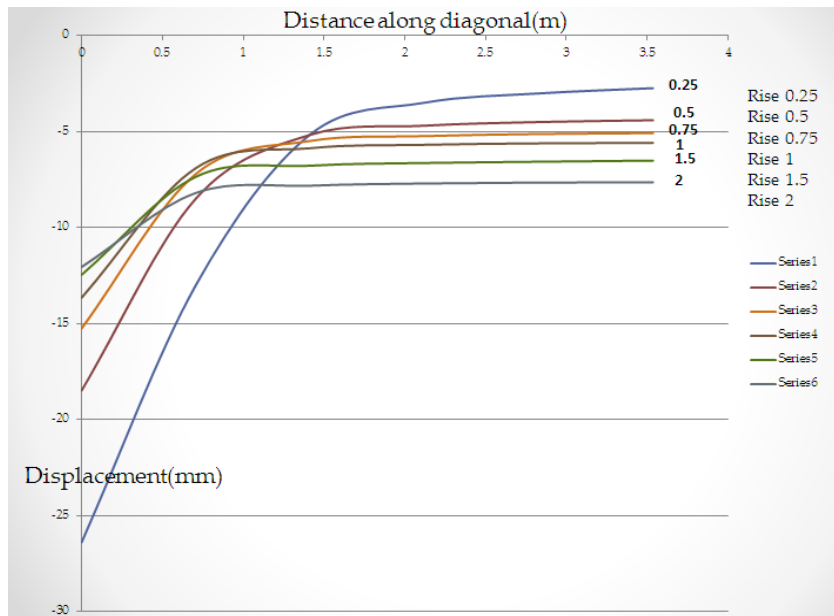
A concentrated load of 1600 KN is assigned at the centre of the shell footing. This load acts as a column load on footing. For the purpose, Nodal load of -1600 KN is assigned at the central node of the shell footing in the local Y direction as shown below:



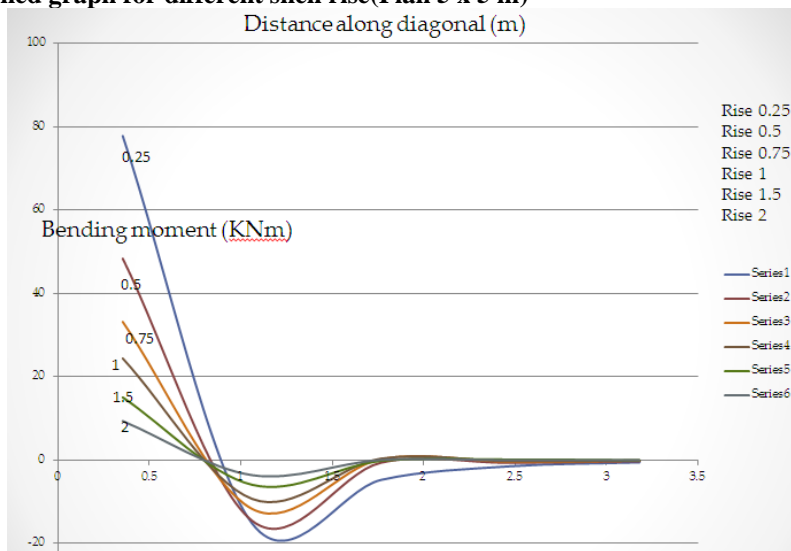
**VI. TEST RESULT**

Multiple Funicular Shell shaped Footings are analyzed with varying Shell Rise, Thickness, Plan and compared for Displacements, Bending Moment and Shear Forces developed.

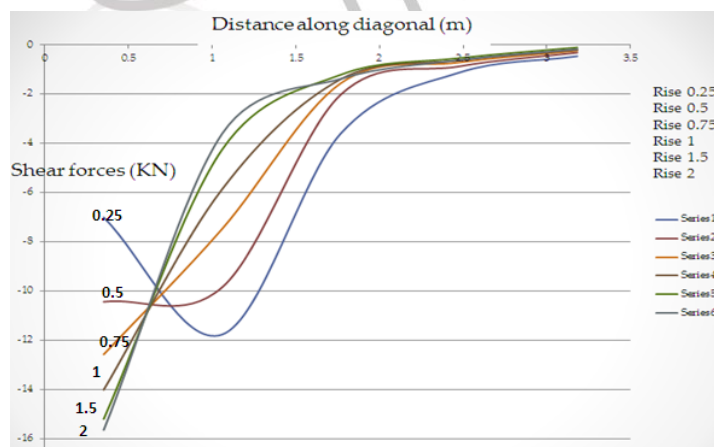
**Displacement-combined graph for different shell rise(Plan 5 x 5 m)**



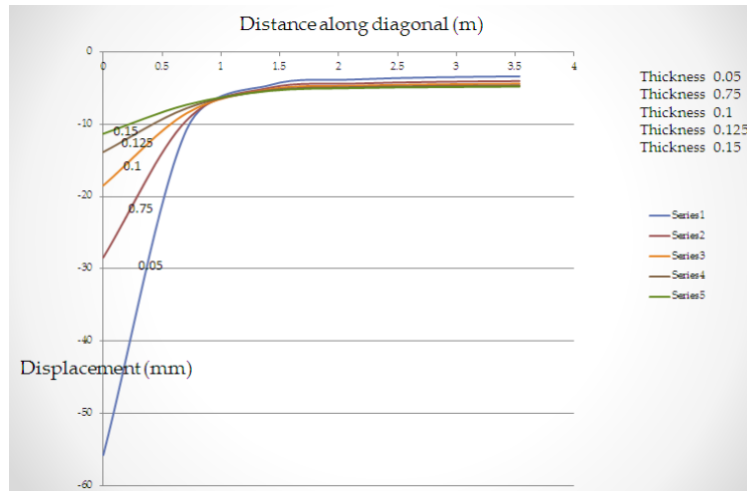
**Bending moment-combined graph for different shell rise(Plan 5 x 5 m)**



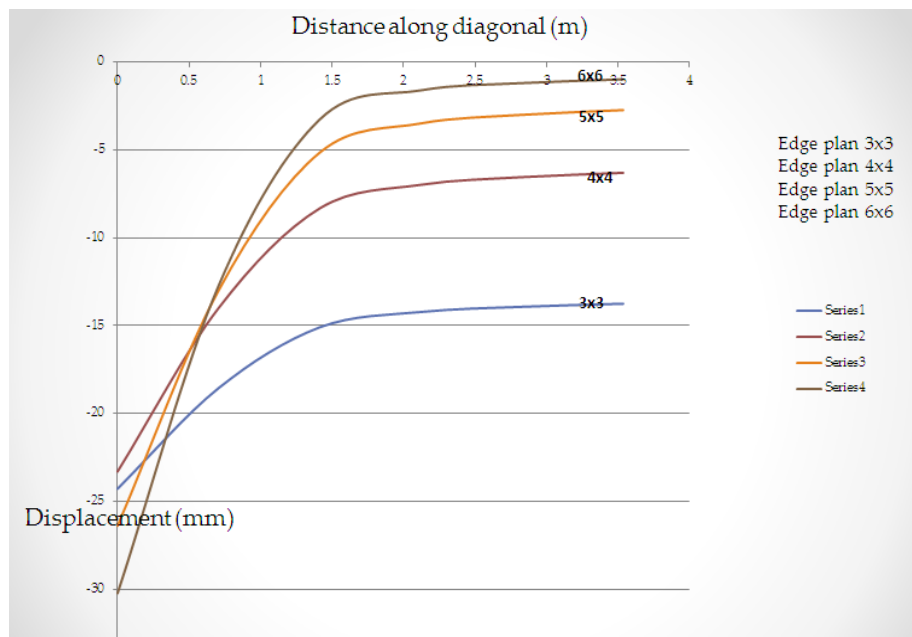
**Shear force-combined graph for different shell rise(Plan 5 x 5 m)**



**Displacement-combined graph for different thickness(Plan 5 x 5 m, rise 0.25m)**



Displacement-combined graph for different edge plan(rise 0.25 m)



**VII. CONCLUSIONS**

Shell footings have been found to be economical foundations in areas having high material to labor cost ratio.

Shell footing has greater load carrying capacity compared with flat shallow foundations. Moreover, shells are essentially thin structures, thus structurally more efficient than flat structures. This is an advantage in situations involving heavy super structural loads to be transmitted to weaker soils

The parametric study on funicular shell foundation depicts the following major conclusions:-

1. Increasing the shell rise from 0.25 m to 2 m.
  - a) Decreases the vertical displacement by about (54.33 %) near and at the center of the shell while it increases by about (178.8 %) when it approaches the edges.
  - b) Decreases the bending moments by about (87.9%).
  - c) Increases the shear forces by about at (123.25%).
2. Increasing shell thickness from 0.025 m to 0.15 m.
  - a) Decreases the vertical displacement by about (80%) at the center of the shell while it increases by about (42.06%) when it approaches the edges.
3. Shell warp should be around 0.04 for better performance.
4. Displacement is inversely proportional to modulus of subgrade reaction.

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