Vedic Mathematics

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Abstract - **This thesis is all about Vedic Mathematics which is the Indian traditional method of calculation. It is composed of 16 Sutras (aphorisms or formulae) and 13 Sub-Sutras (corollaries) that are found in the Atharva Veda which is one of the four Vedas in India. The Sutras (formulae) apply to almost every branch of Mathematics. The application of these Sutras saves a lot of time and effort in solving the problems, compared to the conventional methods of calculations. The applications of these develop one's critical thinking skills and problem-solving skills. The logical proof of these Sutras is explained in detail algebraically which eliminates the misconception that the Vedic Sutras are jugglery.**

***keywords* - Vedic Mathematics**

Chapter 1: General Introduction

Vedic mathematics is an ancient Indian Traditional method of calculations. By using the Vedic mathematics Sutras, any problem can be solved easily, quickly and accurately without using pen and paper even without calculator. Vedic Mathematics calculations are generally based on simple rules and principles of basic arithmetic, algebra, geometry and trigonometry.

Chapter 2:

16 Sutras of Vedic Mathematics and their applications:

1. Ekadhikena Purvena (By one more than the previous one)

For any integer ending with 5, the square always ends with 25 and begins with the product of the previous integer and one more than the integer.

Example:

To find the square of 25, multiply the ten's digit by the immediate next higher number which will turn out to be 2 x 3=6. This is the first part of the answer. To get the second part of the answer simply square the one's digit which is 5x5=25. So the final answer is 625.
Similarly the square of 35 is 1225,
the square of 45 is 2025 and etc.,

Algebraic Proof:

$(ax + b)^2 \equiv a^2 . x^2 + 2abx + b^2$.
Substitute x = 10 and b = 5
$(10a + 5)^2 = a^2 . 10^2 + 2. 10a . 5 + 5^2$
$= a^2 . 10^2 + a.10^2 + 5^2$
$= (a^2 + a) . 10^2 +5^2$
$= a (a + 1) . 10^2 + 25.$
Therefore 10a + 5 is a general form of any two-digit number such as 15, 25, 35, 45, 55,..........,95, 105,115,...... for the values a = 1, 2, 3,............ respectively.

For three digit numbers:

ax^2+bx+c where x =10, a ≠ 0, a, b, c are whole numbers.
$(ax^2 +bx+ c)^2 = a^2 x^4 + b^2 x^2 + c^2 + 2abx^3 + 2bcx + 2cax^2$
$= a^2 x^4 +2ab.x^3 +(b^2 + 2ca)x^2 +2bc . x+ c^2$
Here x = 10 and c = 5
$(a .10^2 + b.10 + 5)^2 = a^2.10^4+2.a.b.10^3 + (b^2+ 2.5.a)10^2+ 2.b.5.10 + 5^2$
$= a^2 .10^4+ 2.a.b.10^3 + (b^2 + 10 a)10^2 + b.10^2+ 5^2$
$= a^2 .10^4+ 2ab.10^3+ b^2 .10^2+ a . 10^3 + b\ 10^2+ 5^2$
$= a^2 .10^4 + (2ab + a).10^3 + (b^2+ b)10^2 +5^2$
$= [a^2 .10^2 +2ab.10 + a.10 + b^2 + b]\ 10^2 + 5^2$
$= (10a + b) (10a+b+1).10^2 + 25$
$= Y (Y+1)\ 10^2+ 25$, where Y = 10a+b.
In general this method works for all numbers that end with 5.

2. Nikhilam Navatascaravam Dasatah:(All from 9 and the last from 10)

To find the product of any two-digit number by any other two-digit number that are close to the base numbers 10,100,1000 etc.,

Example
99 x 96

Solution:
99 -1
96 -4
-1 x -4 = 04
The next step is
99 – 4 or 96 – 1 = 95
95 is the first part and the 04 is the second part of the answer.
Therefore the answer is 9504
We can generalize this method for higher numbers as well.

3. Urdva – Triyagbhyam : (Vertically and crosswise)
To find the product of any two two-digit numbers, the following algorithms can be followed:

Example:
45 x
87
Solution
5 x 7 = 35
(4 x 7) + (5 x 8) = 28 + 40 = 68
4 x 8 = 32
Therefore
45 x 87
= 32 / 68 / 35
= 32 / 68 + 3 / 5
= 32 / 71 / 5
= 32 + 7 / 15
= 3915
Algebraic proof :
Let the two two-digit numbers be (ax+b) and (cx+d).
Here x = 10.
$(ax + b) (cx + d) = ac.x^2 + adx + bcx + b.d$
$= ac.x^2 + (ad + bc)x + b.d$

Algebraic proof for 3 digit numbers:
Let us consider two three-digit numbers
$(ax^2 + bx + c)$ and $(dx^2 + ex + f)$.
Here x=10
Now their product $(ax^2 + bx + c\ x)(dx^2 + ex + f)$
$=ad.x^4+bd.x^3+cd.x^2+ae.x^3+be.x^2+ce.x+af.x^2+bf.x+cf$
$= ad.x^4 + (bd + ae).\ x^3 + (cd + be + af).x^2 + (ce + bf)x + cf$
We can generalize this method for higher-digit numbers.

4. Paraavartya Yojayet (Transpose and Apply)
To divide a large number by a number that is greater than 10 the following method is applied.

Example:
Divide 1225 by 12

Solution:
We need to start from left to right get the answer.
Write the divisor by leaving the ten's digit and write down the unit digit with negative sign and place it below the divisor as shown below.
12
-2

Write down the dividend to the right side as follows.
12 122 5
-2

Write the first digit just below the horizontal line drawn under the dividend and multiply this digit by –2 and write down the product below the second digit and add them.

12 122 5
-2 -2

10

Since 1 x –2 = -2and 2 + (-2) = 0

We get the sum of the second digit is 0. Now multiply the second sum of the second digits by –2 and write down the product under third digit and add them.

12 122 5
- 2 -20
----
102 5

Continue this process until we get the last digit.

12 122 5
- 2 -20 -4
----
102 1

Quotient = 102
Remainder = 1

We can generalize this method for different numbers.

5. Sunyam Samya Samuccaye (When the sum is the same, then the sum is zero)
Example:

14x + 5x…… = 7x + 3x…..
here x = 0.

Example:
18(x+3) = 8(x+3)

Here x+3 is a common factor,
x + 3 = 0
x = -3

Solution by algebraic method:
We can verify the answer now by this conventional method.
18x + 54 = 8x + 24
10x = -30
x = -3

6. Anurupye – Sunyamanyat (If one is in ratio the other one is 0)
Example :
3x + 7y = 2
4x + 21y = 6
Here the ratio of the coefficients of y is 7 : 21 =1 : 3, which is same as the ratio of the independent terms 2 : 6 =1 : 3.
In this case x = 0
Now plug in the value of x=0 in any of the given equations.
21y = 6
7y = 2
y = 2/7

So, the solutions are x=0 and y=2/7

We can generalize this method.

7. Sankalana – Vyavakalanabhyam (By addition and by subtraction)
Let us consider two linear equation in two variables.
ax + by = p
cx + dy = q, where x and y are unknown variables.

The solution is:
$x = (bq - pd) / (bc - ad)$
$y = (cp - aq) / (bc - ad)$

Example:
$1955x - 476y = 2482$
$476x - 1955y = -4913$

Adding both equations:
$2431(x - y) = -2431$
$x - y = -1$

Subtracting both equations:
$1479(x + y) = 7395$
$x + y = 5$
Now solving the new equations
$2x = 4$
$x = 2$
and
$-2y = -6$
$y = 3$

The solution: $x=2$ and $y=3$

8. Puranapuranabhyam (By the completion or non-completion)
This method is called as "the method of completion of polynomials" to find its factors.

Example:

$x^3 + 9x^2 + 24x + 16 = 0$
$x^3 + 9x^2 = -24x - 16$

We know $(x+3)^3 = x^3+9x^2+27x+27$
$=(x^3+9x^2)+27x+27$
$= -24x-16+ 27x+27$
$=3x + 11$ (Substituting above step).

$(x+3)^3 = 3(x+3) + 2$

Let $y = x+3$
Therefore $y^3 = 3y + 2$
$y^3 - 3y - 2 = 0$
Solving, $(y+1)^2 (y-2) = 0$

Solution: $y = -1$ and 2
9. Calana Kalanabhyam (Sequential motion):
This formula is used to solve a quadratic equation by applying calculus and algebra as follows:

Example:
$7x^2 - 11x - 7 = 0$.
Differentiate the equation with respect to x and equate it to the discriminant of the quadratic equation.
$14x - 11 = \pm\sqrt{317}$
Solving for x , we get the solution.

10. Ekanyunena Purvena (One less than the previous) The method is useful in multiplying any number by 9,99,999..

Example:
7 x 9

Solution:
$7 - 1 = 6$
$9 - 6 = 3$

Therefore, 7 X 9 = 63.
This method can be generalized.

11. Yaavadunam (Whatever the extent of its deficiency)
This method is applied to find the square of any number that is closer to multiple of 10.

Example:
$9^2 = [9-(10-9), (10-9) \times (10 – 9)]$
=81

$18^2 = 18 + 8, 8 \times 8$
= 26 / 64
= 26+6 / 4
= 324
This method can be generalized.

12. Vyashtisamanstih (Part and Whole)

Example:
Solve:

$x^3 + 7x^2 + 14x + 8 = 0$
$x^3 + 7x^2 = – 14x – 8$

We know that $(x+3)^3 = x^3+9x^2+27x+27 = 2x^2 + 13x + 19$
Plugging in the above values, we get,

$(x+3)^3= 2x^2 + 13x + 19$
By using Paravartya Sutra,

Dividing $2x^2 + 13x + 19$ by (x+3) we get
$2x^2 + 13x + 19 = (x+3)(2x-7)-2$
i.e. $(x+3)^3 = (x+3)(2x-7)-2$
Let y = x+3
Therefore, $y^3 = y(2y+1) -2$
y = -1, 1, 2

13. Shesanyankena Charamena (The remainders by the last digit)
Example: 1/7

As per the earlier method, the remainders are 1, 3, 2, 6, 4 and 5.
Let's write in the order as 3, 2, 6, 4, 5 and 1.
Multiply them with the last digit of divisor which 7 we get,
21, 14, 42, 28, 35 and 7
Now pick out all the unit digits from the above numbers.
Answer is: 0.142857
This method can be genralized.

14. Sopaantyadvayamantyam (The ultimate and twice the penultimate)
The general form of the equation is:
1/AB + 1/AC = 1/AD + 1/BC,
The solution is: 2C(penultimate) + D(ultimate) = 0

Example:

1/(x+2)(x+3) + 1/(x+2)(x+4) = 1/(x+2)(x+5) + 1/(x+3)(x+4)
2(x+4) + (x+5) = 0
x = -13/3

15. Gunitasamuchyah (The product of the sum is equal to the sum of the product)

Example:

$(x + 3)(x + 2) = x^2 + 5x + 6$
$(1+3)(1+2) = 1 + 5 + 6$
$12 = 12$
Thus verified.

$(x - 4)(2x + 5) = 2x^2 - 3x - 20$
$(1 - 4)(2 + 5) = 2 - 3 - 20$
$-21 = -21$
Thus verified.

16. Gunakasamuchyah (The factors of the sum is equal to the sum of the factors)
For the quadratic equations, the factors of the sum of the coefficients of the variable x in the product is equal to the sum of the coefficients of the variable x in the factors.
13 Sub-sutras of Vedic Mathematics and their applications:
1. Anurupyena (Proportionality)
This Sutra is useful to find product of any two numbers that are nearer to base 10

Example:
46 x 43
Let 50 be the base number. as a base point because it is the nearest powers of base 10.
46 -4
43 -7

Multiplying the difference from 50 to the numbers,
-4 x -7 = 28

43 – 4 = 39 or 46 – 7 = 39
28+50 = 78 which is the last two digits of the answer.

Dividing 39 by 2 which turns out to be with the quotient is19 and with the with remainder 1

46 x 43 = 1978
This method can be generalized.

2. Adyamadyenantya – mantyena (The first by the first and the last by the last)
Example:
$2x^2 + 5x - 3$
Solution:
Find the two numbers such that their product is equal to -6 and their sum is 5.
Therefore the numbers are 6 and -1.
Now write these numbers in such a way that the ratios are equal as follows:
2:6 :: -1:-3
1:3 :: 1:3
Therefore, one of the factors is x+3.
Now
$2x^2 + 5x - 3$
___ ___
x 3
The other factor is 2x-1.

3. Yavadunam Tavadunikrtya Varganca Yojayet (Whatever the deficiency maybe, subtract the deficit and write alongside)
To find the square of any number that is close to the base number 10, we subtract the given number from the base 10 and find the square of the result. Then we subtract the result from the number and do the cross multiplication in order to get the final answer.

Example:
Square of 8
Solution:
10 – 8 = 2,
The square of 2 is 4.
8 – 2 = 6

Thus, Square of 8 = 64
Square of 6

Solution:
10 – 6 = 4,
The square of 4 is 16
6 – 4 = 2
Thus, the square of 6 = 36
This method can be generalized.

4. Antyayor Desakepi
This is a method to calculate the product of two numbers in such a way that the sum of the unit digits is 10.
Example:
42 x 48,
64 x 66,
1325 x 1325 and etc.,

42 x 48 = (4+1) x 4 / 2 x 8
= 5 x 4 / 16
= 20 / 16
= 2016

64 x 66
= (6 + 1) x 6 / 4 x 6
= 7 x6 / 24
= 4224

1125 x 1125
= 113 x 112 / 5 x 5
= 12656 / 25
= 1265625
This method can be generalized.

5. Antyayoreva (Calculation with only the last terms)
The general format of the equation of this type is:
(AC + D) / (BC + E) = A/B
The solution is:
A/B = D/E

Example:
$(x^2+x+1) / (x^2+3x+3) = (x+1) / (x+3)$
$[x(x+1)+1] / [x(x+3)+1] = (x+1) / (x+3)$
$(x+1) / (x+3) = 1 / 3$
$x = 0$

6. Lopana Sthapanabhyam (By the method of alternate elimination and retention)
Example:
Factorize: $2x^2 + 6y^2 + 3z^2 + 7xy + 11yz + 7zx$
where x, y, z. are three variables.
Eliminate the variable z by setting it to 0, z=0.
Expression = $2x^2+ 6y^2+ 7xy$
= (x+2y) (2x+3y
Now substitute, y=0, then
Expression = $2x^2+ 3z^2+ 7zx$
= (x+3z) (2x+z)

Expression =(x+2y+3z)(2x+3y+z)

7. Vilokanam (By mere observation)
We can solve the problems by mere observation.
x + 1/x = 10/3
x + 1/x = 3 + 1/3
x = 3, 1/3

This method can be generalized.

8. Gunitahsamuccayah Samuccayagunitah
This method is applied to verify the roots of the quadratic equation.

Example:
(x + 3) (x + 2) = x^2 + 5x + 6
(1+3) (1+2) = 1 + 5 + 6
12 = 12
Hence verified.
This method can be generalized.

9. Sisyate Sesasamjnah (The remainder remains constant)
To find the product of any two numbers that are nearer to the number more than multiple of 10.
Example:
102x104
Solution:
102 +2
104 +4

102+4 or 104+2 = 106
2x4=08
The answer is 10608
This method can be generalized.
10. Gunakasamuccayah
For quadratic equation, the factor of the sum of the coefficients of 'x' in the product is equal to the sum of the coefficients of 'x' in the factors.

11. Vestanam (By Osculation method)
This method is used to check for divisibility of a number.Let us check
Example:
Check if 21 is divisible by 7

Solution:
1×5+2 = 7
Therefore 21 is divisible by 7.

91
1×5+9 = 14 which is a multiple of 7.
Therefore 91 is divisible by 7.
This method can be generalized.

12. Yavadunam Tavadunam
Solving simultaneous equations with two variables:

ax + by = p
cx + dy = q

The solution is:
x = (bq – pd) / (bc – ad)
y = (cp – aq) / (bc – ad)

Example:
2x + 3y =6
3x + 4y = 3

The solution is:
x = (9 – 24)/ (9 – 8) = -15
y = (18 – 6) (9 – 8) = 12
This method can be generalized.

13. Kevalaih Saptakam Gunyat.

For the number 7 the multiplicand is 143 (Kevala:143, Sapta:7)
Using the result 1/7=0.142857, we can calculate the following:
1/7 = 0.142857
By Ekanyuna Sutra,
143 x 999 = 142857

2/7 = 0.285714
3/7 = 0.428571
4/7 = 0.571428
5/7 = 0.714285
6/7 = 0.857142
All of the above values are in cyclic order.
Tips to remember the last digits:
2/7: the last digit of answer must be 4 (2*7=14)
3/7: the last digit of answer must be 1 (3*7=21)
4/7: the last digit of answer must be 8 (4*7=28)
5/7: the last digit of answer must be 5 (5*7=35)
6/7: the last digit of answer must be 2 (6*7=42)

This method can be generalized.

Conclusion

These Vedic Mathematics methods discussed in this thesis are intended for any reader with some basic mathematical background. However more steps are needed to touch the latest developments in Vedic Mathematics. I was able to cover the major parts of Vedic Mathematics in this thesis. I feel that any reader can enjoy the diversity and simplicity in applying Vedic Mathematics more easier than applying conventional methods.

References:

LIST OF SUTRAS (FORMULAE)

- Ekadhiken Purvena (By one more than the previous one)
- Nikhilam Navatashcharamam Dashatah (All from nine and last from ten)
- Urdhva tirgbhyam (Vertically and crosswise)
- Paravartya Yojayet (Transpose and apply)
- Sunyam Samyasamuchchaye (The summation is equal to zero)
- Anurupye Sunyamanyat (If one is in ratio, the other is zero)
- Sankalana Vyavakalanabhyam (By addition and subtraction)
- Puranapuranabhyam (By the completion and non-completion)
- Chalana kalanabhyam (Sequential motion)
- Yavadunam (The deficiency)
- Vyashtisamashtih (Whole as one and one as whole)
- Sheshanyankena charamena (Remainder by the last digit)
- Sopantyadvayamantyam (Ultimate and twice penultimate)
- Ekanyunena Purvena (By one less than the previous one)
- Gunitasamuchchayah (The whole product is the same)
- Gunakasamuchchayah (Collectivity of multipliers)

Appendix

Vedic Mathematics:

Student Feedback Questionnaire:

Student Name:----------------------------------
Date: ------------------------------ -----------
Vedic Mathematics Topic: --------------
Please take a moment to answer the following questions:
1. Were you able to complete assignments during class time?
YES NO
2. Were you able to complete the homework assignments?
YES NO
3. Circle the sentence that best describes how many assignment questions you understood.
a. I understood none of the questions
b. I understood some of the questions
c. I understood most of the questions
d. I understood all of the questions

4. Circle words that best describe how you felt about this Vedic Mathematics topic.
Frustrated, angry, confused, indifferent, happy, successful, confident
5. Do you have other comments?
If so write here..........